

# 9

## *Electrostatic Accelerators and Pulsed High Voltage*

In this chapter we begin the study of charged particle acceleration. Subsequent chapters describe methods for generating high-energy charged particle beams. The kinetic energy of a charged particle is increased by electric fields according to  $\Delta T = \int \mathbf{E}(\mathbf{x},t) \cdot d\mathbf{x}$  [Eq. (3.16)], where the integral is taken along the particle orbit. This equation applies to all accelerators. The types of accelerators discussed in the next six chapters differ by the origin and characteristics of the electric field. In the most general case,  $\mathbf{E}$  varies in time and position. In this chapter, we limit consideration to the special case of static electric fields.

Static electric fields can be derived from a potential function with no contribution from time-varying magnetic flux. An electrostatic accelerator consists basically of two conducting surfaces with a large voltage difference  $V_o$ . A particle with charge  $q$  gains a kinetic energy  $qV_o$ . In this chapter, we shall concentrate mainly on methods for generating high voltage. The description of voltage sources, particularly pulsed voltage generators, relies heavily on passive circuit analysis. Section 9.1 reviews the properties of resistors, capacitors, and inductors. A discussion of circuits to generate dc voltage follows in Section 9.2. The equations to describe ideal transformers are emphasized. A thorough knowledge of the transformer is essential to understand linear induction accelerators (Chapter 10) and betatrons (Chapter 11).

Section 9.4 introduces the Van de Graaff voltage generator. This device is used extensively in low-energy nuclear physics. It also provides preacceleration for beams on many high-energy accelerators. The principles of dc accelerators are easily understood - the main difficulties associated with these machines are related to technology, which is largely an empirical field. Properties of insulators are discussed in Section 9.3 and Paschen's law for spark breakdown of an

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insulating gas is derived. The important subject of vacuum breakdown is reviewed in Section 9.5.

The remainder of the chapter is devoted to techniques of pulsed voltage generation. Acceleration by pulsed voltage accelerators is well described by the electrostatic approximation because the transit time of particles and the propagation time for electromagnetic waves in acceleration gaps are small compared to typical voltage pulslengths ( $\Delta t \geq 50$  ns). Although the static approximation describes the acceleration gap, the operation of many pulsed power circuits, such as the transmission line, involves electromagnetic wave propagation. Pulsed voltage generators have widespread use in accelerators characterized by cyclic operation. In some instances they provide the primary power, such as in high-current relativistic electron beam generators and in the linear induction accelerator. In other cases, they are used for power conditioning, such as klystron drivers in high-gradient rf electron linear accelerators. Finally, pulsed voltage modulators are necessary to drive pulsed extraction and injection fields for synchrotrons and storage rings.

Pulsed voltage circuits must not only produce a high voltage but must also shape the voltage in time. Sections 9.5-9.14 introduce many of the circuits and techniques used. Material is included on critically damped circuits (Section 9.6) impulse generators (Section 9.7), transmission line modulators (Section 9.9), the Blumlein transmission line (Section 9.10), pulse forming networks (Section 9.11), power compression systems (Section 9.12), and saturable core magnetic switching (Section 9.13). The equations for electromagnetic wave propagation in a transmission line are derived in Section 9.8. This section introduces the principles of interacting electric and magnetic fields that vary in time. The results will be useful when we study electromagnetic wave phenomena in rf accelerators. Material is also included on basic methods to measure fast-pulsed voltage and current (Section 9.14).

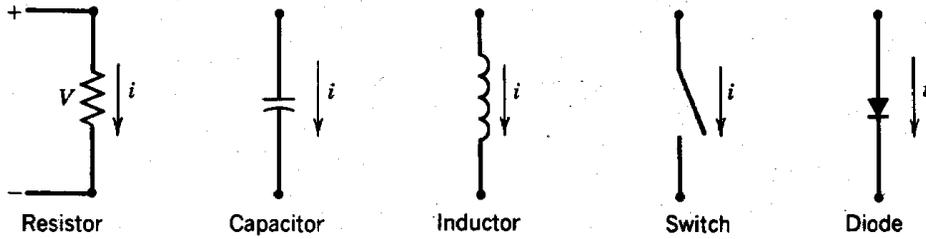
## 9.1 RESISTORS, CAPACITORS AND INDUCTORS

High-voltage circuits can usually be analyzed in terms of five elements: resistors, capacitors, inductors, ideal diodes, and ideal switches. These elements are two-terminal passive components. Two conductors enter the device, and two quantities characterize it. These are the current through the device and the voltage between the terminals. The action of the circuit element is described by the relationship between voltage and current. Standard symbols and polarity conventions are shown in Figure 9.1.

### A. Voltage-Current Relationships

The ideal diode has zero voltage across the terminals when the current is positive and passes zero current when the voltage is negative. Diodes are used for rectification, the conversion of a bipolar voltage waveform into a dc voltage. A switch is either an open circuit (zero current at any

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**Figure 9.1** Symbols and polarity conventions for common circuit elements.

voltage) or a closed circuit (zero voltage at any current). In pulsed voltage circuits, a closing switch is an open circuit for times  $t < 0$  and a short circuit for  $t > 0$ . An opening switch has the inverse properties. The spontaneous breakdown gap used on some pulse power generators and pulse sharpening circuits is an example of a two-terminal closing switch. Triggered switches such as the thyatron and silicon-controlled rectifier are more common; they are three terminal, two-state devices.

In this section, we consider the properties of resistors, capacitors, and inductors in the *time-domain*. This means that time variations of the total voltage are related to time variations of the total current. Relationships are expressed as differential equations. It is also possible to treat circuits in the *frequency-domain*. This approach is useful for oscillating harmonic circuits, and will be applied to rf cavities in Chapter 12. In the frequency-domain analysis, time variations are Fourier analyzed in terms of the angular frequency  $\omega$ . Each harmonic component is treated separately. In the frequency-domain analysis, voltage and current in individual elements and multielement circuits are related simply by  $V = a(\omega)I$ , where  $a(\omega)$  may be a complex number.

A resistor contains material that impedes the flow of electrons via collisions. The flow of current is proportional to the driving voltage

$$I = V/R, \tag{9.1}$$

where  $I$  is in amperes,  $V$  in volts, and  $R$  is the resistance in ohms ( $\Omega$ ). Energy is transferred from flowing electrons to the resistive material. With the polarity shown in Figure 9.1, electrons flow into the bottom of the resistor. Each electron absorbs an energy  $eV_o$  from the driving circuit during its transit through the resistor. This energy acts to accelerate the electrons between collisions. They emerge from the top of the resistor with low velocity because most of the energy gained was transferred to the material as heat. The number of electrons passing through the resistor per second is  $I/e$ . The power deposited is

$$P \text{ (watts)} = VI = V^2/R = I^2R. \tag{9.2}$$

As we saw in Section 5.2, the basic capacitor geometry has two conducting plates separated by a dielectric (Figure 5.6c). The voltage between the plates is proportional to the stored charge on the plates and the geometry of the capacitor:

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$$V = Q/C. \quad (9.3)$$

The quantity  $V$  is in volts,  $Q$  is in coulombs, and  $C$  is in farads (F). Neglecting fringing fields, the capacitance of the parallel plate geometry can be determined from Eq. (3.9):

$$C = \frac{\epsilon_0 (\epsilon/\epsilon_0) A}{\delta} \quad (F), \quad (9.4)$$

where  $\epsilon_0 = 8.85 \times 10^{-12}$  and  $\epsilon/\epsilon_0$  is the relative dielectric constant of the material between the plates,  $A$  is the plate area in square meters, and  $\delta$  is the plate spacing in meters. Small capacitors in the pF range ( $10^{-12}$  F) look much like Figure 5.6c with a dielectric such as Mylar ( $\epsilon/\epsilon_0 \sim 2-3$ ). High values of capacitance are achieved by combining convoluted reentrant geometries (for large  $A$ ) with high dielectric constant materials. The current through a capacitor is the time rate of change of the stored charge. The derivative of Eq. (9.3) gives

$$I = C (dV/dt). \quad (9.5)$$

The capacitor contains a region of electric field. The inductor is configured to produce magnetic field. The most common geometry is the solenoidal winding (Fig. 4.18). The magnetic flux linking the windings is proportional to the current in the winding. The voltage across the terminals is proportional to the time-rate of change of magnetic flux. Therefore,

$$V = L (di/dt), \quad (9.6)$$

where  $L$  is a constant dependent on inductor geometry. Inductance is measured in Henries (H) in the mks system.

### B. Electrical Energy Storage

A resistor converts electrical to thermal energy. There is no stored electrical energy that remains in a resistor when the voltage supply is turned off. Capacitors and inductors, on the other hand, store electrical energy in the form of electric and magnetic fields. Electrical energy can be extracted at a latter time to perform work. Capacitors and inductors are called *reactive* elements (*i.e.*, the energy can act again).

We can prove that there is no average energy lost to a reactive element from any periodic voltage waveform input. Voltage and current through the element can be resolved into Fourier components. Equations (9.5) and (9.6) imply that the voltage and current of any harmonic component are  $90^\circ$  out of phase. In other words, if the voltage varies as  $V_o \cos(\omega t)$ , the current varies as  $\pm I_o \sin(\omega t)$ . Extending the arguments leading to Eq. (9.2) to reactive elements, the total energy change in an element is

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$$\Delta U = \int P dt = I_0 V_0 \int \sin(\omega t) \cos(\omega t).$$

Although the energy content of a reactive element may change over an oscillation period, the average over many periods is zero.

Energy is stored in a capacitor in the form of electric fields. Multiplying Eq. (5.19) by the volume of a parallel-plate capacitor, the stored energy is

$$U_c = \frac{1}{2} [\epsilon_0 (\epsilon/\epsilon_0) (V/\delta)^2] A\delta. \quad (9.7)$$

Comparing Eq. (9.7) to (9.4),

$$U_c = \frac{1}{2} CV^2. \quad (9.8)$$

The magnetic energy stored in an inductor is

$$U_c = \frac{1}{2} LI^2. \quad (9.9)$$

### C. Common Capacitor and Inductor Geometries

The coaxial capacitor (Fig. 9.2a) is often used as an energy storage device for high-voltage pulsed power generators. The electrodes are cylinders of length  $d$  with radii  $R_o$  and  $R_i$ . The cylinders have a voltage difference  $V_o$ , and there is a medium with relative dielectric constant  $(\epsilon/\epsilon_0)$  between them. Neglecting fringing fields at the ends, the electric field in the dielectric region is

$$E_r = Q/2\pi\epsilon rd. \quad (9.10)$$

The integral of Eq. (9.10) from  $R_i$  to  $R_o$  equals  $V_o$  or

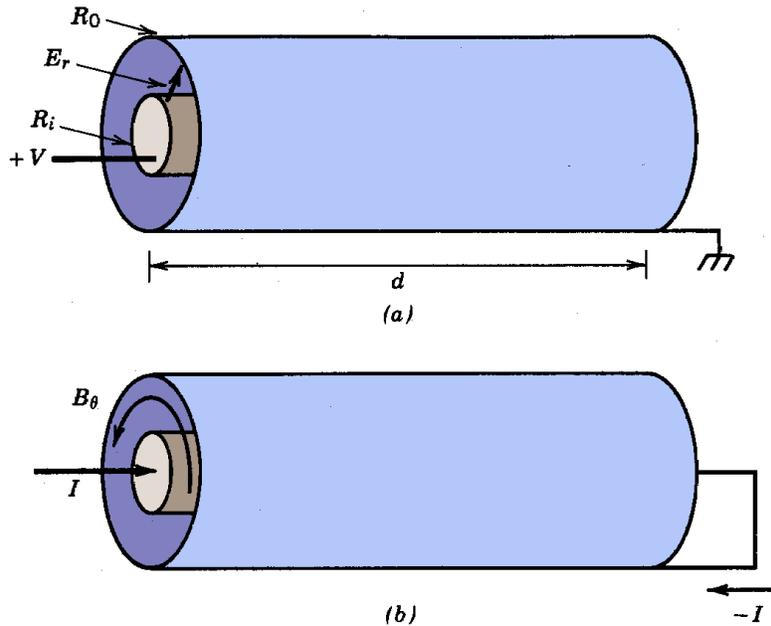
$$V_o = (Q/2\pi\epsilon d) \ln(R_o/R_i). \quad (9.11)$$

Comparing Eq. (9.11) with (9.3) shows the capacitance is

$$C = 2\pi\epsilon d / \ln(R_o/R_i). \quad (9.12)$$

All current-carrying elements produce magnetic fields and thus have an inductance. There is a magnetic field between the cylinders of Figure 9.2b if current flows along the center conductor.

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**Figure 9.2** Lumped circuit elements. (a) Coaxial capacitor. (b) Coaxial inductor.

An equal and opposite current must return along the outer conductor to complete the circuit; therefore, there is no magnetic field for  $r > R_o$ . Equation (4.40) specifies that the field between the cylinders is

$$B_\theta(r) = \mu_0 I / 2\pi r \quad (9.13)$$

if there is no ferromagnetic material in the intercylinder volume. The total magnetic field energy is

$$U_m = d \int dr (2\pi r) (B_\theta^2 / 2\mu_0) = (\mu_0 / 2\pi) (I^2 / 2) (d \ln(R_o / R_i)). \quad (9.14)$$

Setting  $U_m$  equal to  $LI^2/2$  implies that

$$L = (\mu_0 / 2\pi) d \ln(R_o / R_i). \quad (9.15)$$

Equation (9.15) should be multiplied by  $\mu/\mu_0$  if the coaxial region contains a ferromagnetic material with an approximately linear response.

Coaxial inductors generally have between 0.1 and 1  $\mu\text{H}$  per meter. Higher inductances are produced with the solenoidal geometry discussed in Section 4.6. We shall make the following assumptions to calculate the inductance of a solenoid: edge fields and curvature effects are neglected, the winding is completely filled with a linear ferromagnetic material, and  $N$  series windings of cross-sectional area  $A$  are uniformly spaced along a distance  $d$ . The magnetic

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field inside the winding for a series current  $I$  is

$$B_z = \mu I (N/d) \quad (9.16)$$

according to Eq. (4.42). Faraday's law implies that a time variation of magnetic field linking the windings produces a voltage

$$V = NA (dB_z/dt). \quad (9.17)$$

We can identify the inductance by combining Eqs. (9.16) and (9.17):

$$L = \mu_0 (\mu/\mu_0) N^2 A/d. \quad (9.18)$$

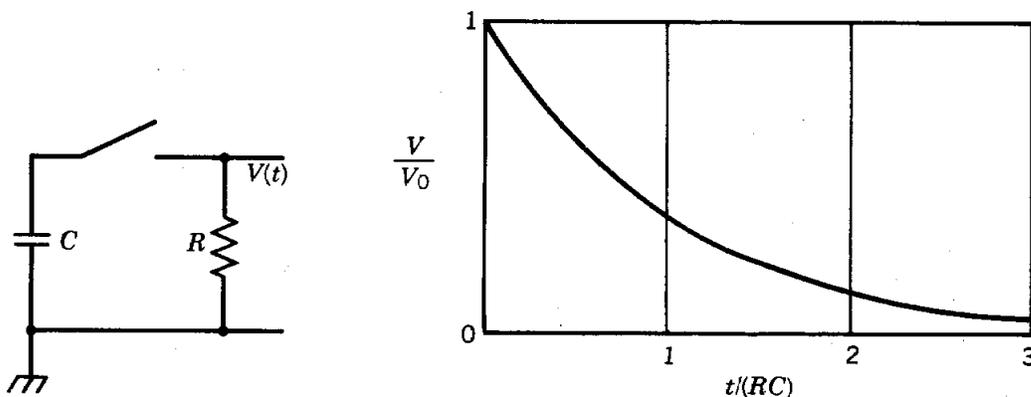
Compact solenoids can assume a wide range of inductance values because of the strong scaling with  $N$ .

### D. Introductory Circuits

Figure 9.3 shows a familiar circuit combining resistance, capacitance, and a switch. This is the simplest model for a pulsed voltage circuit; electrical energy is stored in a capacitor and then dumped into a load resistor via a switch. Continuity of current around the circuit combined with Eqs. (9.1) and (9.5) implies the following differential equation for the load voltage after switching:

$$C (dV/dt) + V/R = 0. \quad (9.19)$$

The solution (plotted in Fig. 9.3) for switching at  $t = 0$  is  $V(t) = V_0 \exp(-t/RC)$  with  $V_0$  the initial



**Figure 9.3** Time-dependent load voltage in a switched  $RC$  circuit.

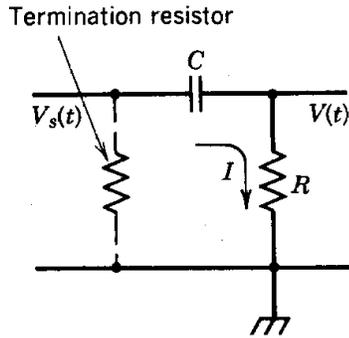


Figure 9.4 Passive integrator.

charge voltage. The  $RC$  time ( $\Delta t = RC$ ) is the characteristic time for the transfer of energy from a capacitor to a resistor in the absence of inductance.

The passive integrator (Fig. 9.4) is a useful variant of the  $RC$  circuit. The circuit can integrate fast signals in the nanosecond range. It is often used with the fast diagnostics described in Section 9.14. Assume that voltage from a diagnostic,  $V(t)$  is incident from a terminated transmission line (Section 9.10). The signal has duration  $\Delta t$ . When  $\Delta T \ll RC$ , the voltage across the capacitor is small compared to the voltage across the resistor. Thus, current flowing into the circuit is limited mainly by the resistor and is given by  $I \approx V_s(t)/R$ . Applying Eq. (9.5), the output voltage is

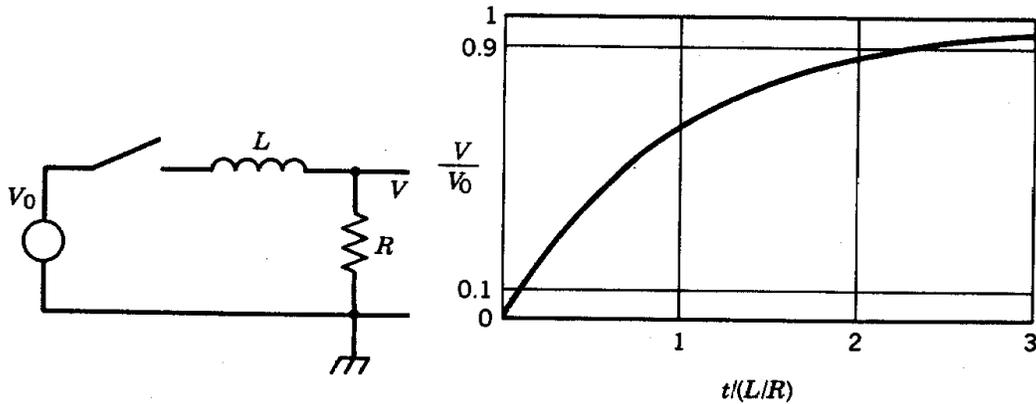
$$V_{out} = \int V_s(t) dt / RC. \quad (9.20)$$

The passive integrator is fast, resistant to noise, and simple to build compared to a corresponding circuit with an operational amplifier. The main disadvantage is that there is a droop of the signal. For instance, if  $V_s$  is a square pulse, the signal at the end of the pulse is low by a factor  $1 - \Delta t/RC$ . An accurate signal integration requires that  $RC \gg \Delta t$ . This condition means that the output signal is reduced greatly, but this is usually not a concern for the large signals available from fast diagnostics.

A circuit with an inductor, resistor, voltage source, and switch is shown in Figure 9.5. This circuit models the output region of a pulsed voltage generator when the inductance of the leads and the load is significant. Usually, we want a rapid risetime for power into the load. The time for initiation of current flow to the load is limited by the undesirable (or parasitic) inductance. Continuity of current and the V-I relations of the components [Eqs. (9.1) and (9.6)] give the following differential equation for the load voltage:

$$\int V dt / L = V/R = 0. \quad (9.21)$$

The time variation for load voltage, plotted in Figure 9.5, is



**Figure 9.5** Time-dependent load voltage from a pulse generator with a series inductance.

$$V(t) = V_0 [1 - \exp(-t/(L/R))]. \quad (9.22)$$

The  $L/R$  time determines how fast current and voltage can be induced in the load. The 10-90% risetime for the voltage pulse is  $2.2(L/R)$ . The load power varies as

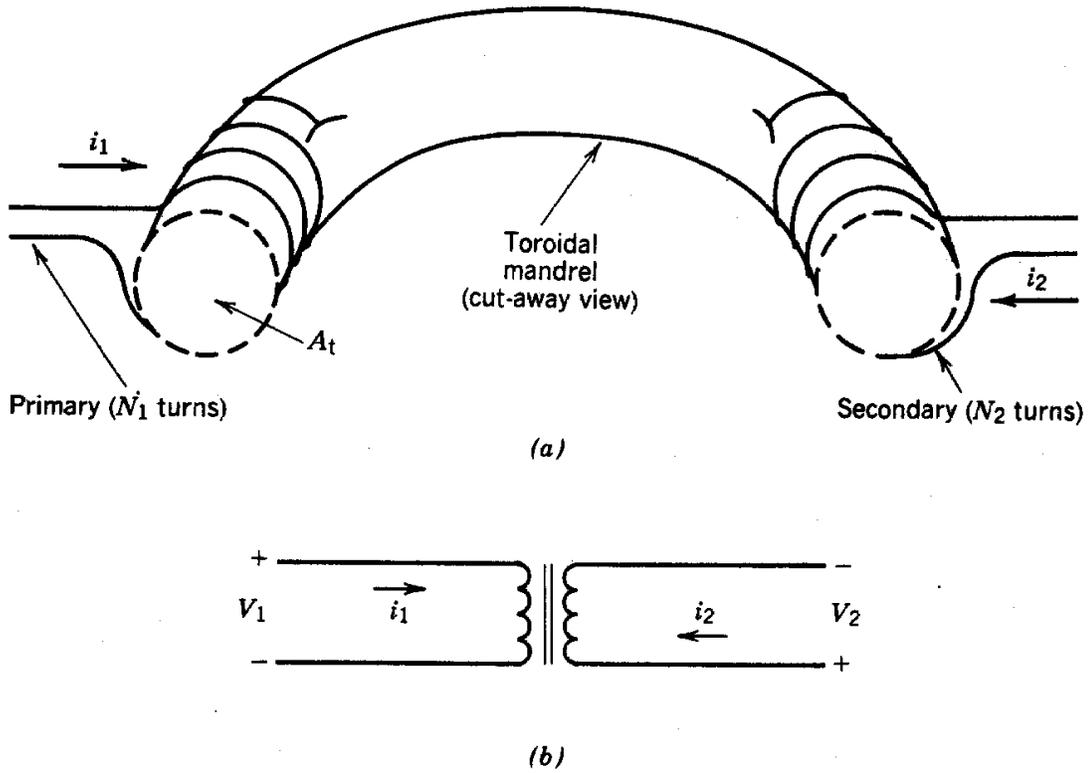
$$P(t) = V^2(t)/R = (V_0^2/R) [1 - \exp(-t/(L/R))]^2. \quad (9.23)$$

The 10-90 time for the power pulse is  $2.6(L/R)$ . As an example, if we had a 50-ns pulse generator to drive a  $25 \Omega$  load, the total inductance of the load circuit must be less than  $0.12 \mu\text{H}$  if the risetime is to be less than 25% of the pulsewidth.

## 9.2 HIGH-VOLTAGE SUPPLIES

The transformer is a prime component in all high-voltage supplies. It utilizes magnetic coupling to convert a low-voltage ac input to a high-voltage ac output at reduced current. The transformer does not produce energy. The product of voltage times current at the output is equal to or less than that at the input. The output can be rectified for dc voltage.

We will first consider the air core transformer illustrated in Figure 9.6. Insulated wire is wound uniformly on a toroidal insulating mandrel with  $\mu \approx \mu_0$ . There are two overlapped windings, the primary and the secondary. Power is introduced in the primary and extracted from the secondary. There is no direct connection between them; coupling is inductive via the shared magnetic flux in the torus. Assume that the windings have cross-sectional area  $A_t$  and average radius  $r_t$ . The primary winding has  $N_1$  turns, and the secondary winding has  $N_2$  turns. The symbol for the transformer with polarity conventions is indicated in Figure 9.6. The windings are oriented so that



**Figure 9.6** Air core transformer. (a) Geometry. (b) Circuit symbol.

positive  $i_1$  and positive  $i_2$  produce magnetic fields in opposite directions.

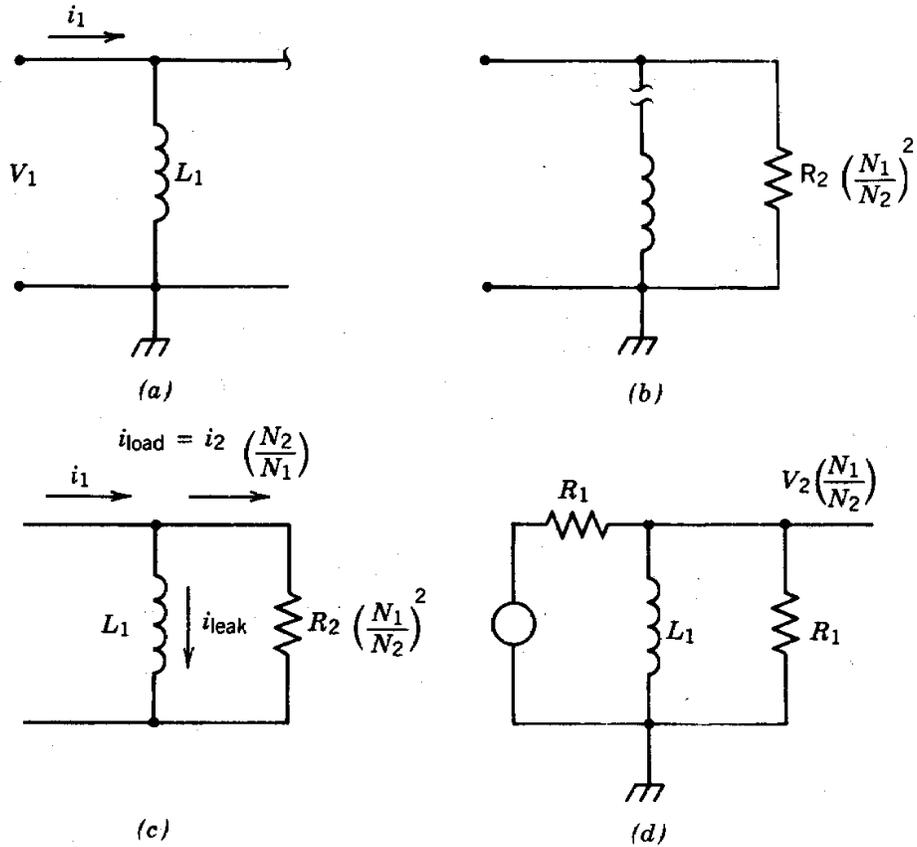
We will determine  $V_2$  and  $i_2$  in terms of  $V_1$  and  $i_1$  and find a simple model for a transformer in terms of the circuit elements of Section 9.1. First, note that energy entering the transformer on the primary can have two destinations. It can be transferred to the secondary or it can produce magnetic fields in the torus. The ideal transformer transfers all input energy to a load connected to the secondary; therefore, the second process is undesirable. We will consider each of the destinations separately and then make a combined circuit model.

To begin, assume that the secondary is connected to an open circuit so that  $i_2 = 0$ , no energy is transferred to the secondary. At the primary, the transformer appears to be an inductor with  $L_1 = \mu_0 N_1^2 A_t / 2\pi r_t$ . All the input energy is converted to magnetic fields; the load is reactive. The equivalent circuit is shown in Figure 9.7a.

Next, suppose the secondary is connected to a resistive load and that there is a way to make  $L_1$  infinitely large. In this case, there is no magnetic field energy and all energy is transferred to the load. Infinite inductance means there are no magnetic fields in the torus with finite driving voltage. The field produced by current flow in the primary is exactly canceled by the secondary field. This means that

$$N_1 i_1 \cong N_2 i_2. \tag{9.24}$$

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**Figure 9.7** Equivalent circuit models for a transformer with ideal coupling. (a) Open circuit load on secondary. (b) Infinite primary shunt inductance. (c) Basic circuit model for an ideal transformer with a resistive load on the secondary. (d) Driving a resistive load on the secondary by a pulse modulator with matched impedance at the primary.

Equation (9.24) holds, if the transformer is perfectly wound so that the primary and secondary windings enclose the same area. This situation is called *ideal coupling*. With ideal coupling, the windings enclose equal magnetic flux so that

$$\begin{aligned} N_1 (d\Phi/dt) &= V_1, \\ N_2 (d\Phi/dt) &= V_2. \end{aligned} \tag{9.25}$$

and hence,

$$V_2 = V_1 (N_2/N_1). \tag{9.26}$$

Equations (9.24)-(9.26) indicate that energy exchange between the windings is through a changing magnetic flux and that there is a voltage step-up when  $N_2 > N_1$ . In the case of infinite  $L_1$

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the combination of Equations 9.24 and 9.26 gives the condition for conservation of energy,

$$V_1 i_1 = V_2 i_2. \quad (9.27)$$

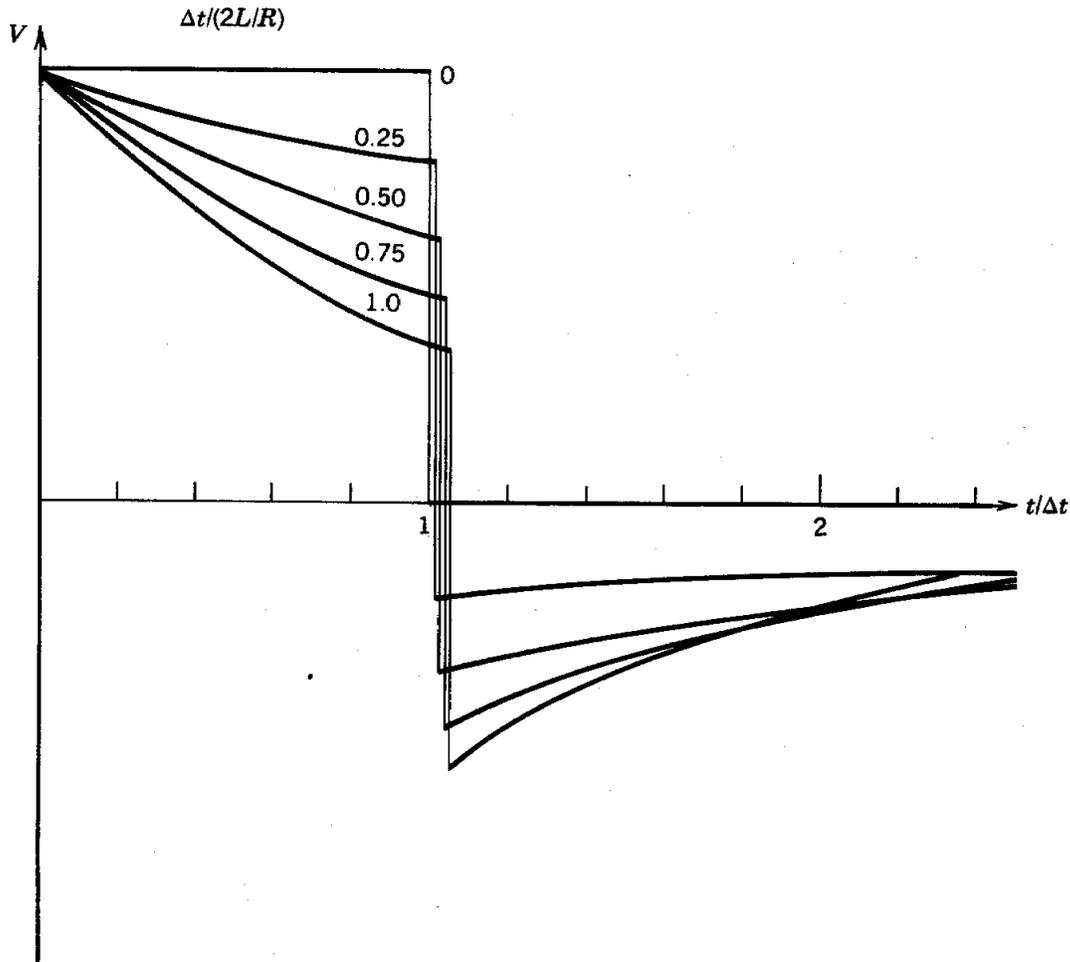
When the secondary is connected to a load resistor  $R_2$ , the primary voltage is proportional to current via  $V_1 = i_1 R_2 (N_1/N_2)^2$ . Thus, when viewed from the primary, the circuit is that illustrated in Figure 9.7b with a transformed load resistance.

We have found voltage-current relationships for the two energy paths with the alternate path assumed to be an open circuit. The total circuit (Fig. 9.7c) is the parallel combination of the two. The model shown determines the primary current in terms of the input voltage when the secondary is connected to a resistive load. The input current is expended partly to produce magnetic field in the transformer ( $i_{leak}$ ) with the remainder coupled to the secondary ( $i_{load}$ ). The secondary voltage is given by Eq. (9.26). The secondary current is given by Eq. (9.27) with  $i_{load}$  substituted for  $i_l$ . The conclusion is that if the primary inductance is low, a significant fraction of the primary current flows in the reactance; therefore  $i_1 > (N_2/N_1) i_1$ .

The reactive current component is generally undesirable in a power circuit. The extra current increases resistive losses in the transformer windings and the ac voltage source. If the transformer is used to amplify the voltage of a square pulse from a pulsed voltage generator (a common accelerator application), then leakage currents contribute to droop of the output voltage waveform. Consider applying a square voltage pulse of duration  $\Delta t$  from a voltage generator with an output impedance  $R_1 = R_2 (N_1/N_2)^2$ . The equivalent circuit has resistance  $R_l$  in series with the primary (Fig. 9.7d). We will see in Section 9.10 that this is a good model for the output of a charged transmission line. The output pulse shape is plotted in Figure 9.8 as a function of the ratio of the circuit time  $L/[R_2(N_1/N_2)^2]$  to the pulselength  $\Delta t$ . The output pulse is a square pulse when  $L/R_1 \gg \Delta t$ . If this condition is not met, the pulse droops. Energy remains in transformer magnetic fields at the end of the main pulse; this energy appears as a negative post-pulse. Although no energy is lost in the ideal transformer, the negative post-pulse is generally useless. Therefore, pulse transformers with low primary inductance have poor energy transfer efficiency and a variable voltage output waveform. Drooping waveforms are often unacceptable for accelerator applications.

Leakage current is reduced by increasing the primary inductance. This is accomplished by constructing the toroidal mandrel from ferromagnetic material. The primary inductance is increased by a factor of  $\mu/\mu_0$ . Depending on the operating regime of the transformer, this factor may be as high as 10,000. Another way to understand the role of iron in a transformer is to note that a certain flux change is necessary to couple the primary voltage to the secondary. In an air core transformer, the flux change arises from the difference in the ampere turns between the primary and secondary; the flux change is generated by the leakage current. When a ferromagnetic material is added, atomic currents contribute to the flux change so that the leakage current can be much smaller.

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**Figure 9.8** Pulse waveforms; resistive load driven by a matched pulse modulator through a transformer:  $\Delta t$  is the duration of the voltage pulse,  $R$  is the output impedance of the modulator and the load resistance viewed at the primary, and  $L$  is the primary shunt inductance.

The energy transfer efficiency of real transformers falls short of that for the ideal transformer of Figure 9.7c. One reason for power loss is that ferromagnetic materials are not ideal inductors. Eddy current losses and hysteresis losses in iron transformer cores for fast pulses are discussed in Section 10.2. Another problem is that ideal coupling between the primary and secondary cannot be achieved. There is a region between the windings in which there is magnetic field that is not canceled when both windings have the same ampere turns; the effect of this region is represented as a series inductance in the primary or secondary. Minimizing the effects of nonideal coupling is one of the motivations for using an iron core to increase the primary inductance of the transformer rather than simply increasing the number of turns in the primary and secondary windings of an air core transformer.

There are limitations on the primary voltage waveforms that can be handled by pulse transformers with ferromagnetic cores. The rate of change of flux enclosed by the primary is given

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by Eq. (9.25a). The flux enclosed in a transformer with a toroidal core area  $A_l$  is  $A_l B(t)$ . Inserting this into Eq. (9.25a) and integrating, we find that

$$[B(t) - B(0)] N_1 A_l = \int V(t) dt. \quad (9.28)$$

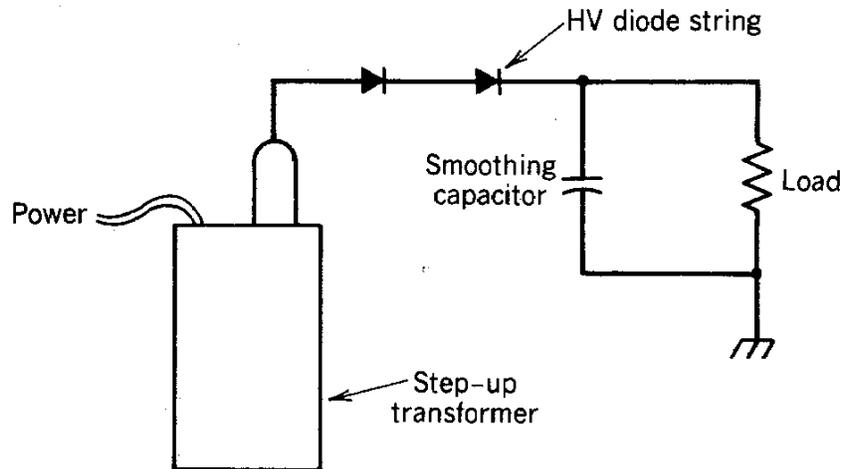
Equation (9.28) constrains the input voltage in terms of the core geometry and magnetic properties of the core material. An input signal at high voltage or low frequency may drive the core to saturation. If saturation occurs, the primary inductance drops to the air core value. In this case, the primary impedance drops, terminating energy transfer to the load. Referring to the hysteresis curve of Figure 5.12, the maximum change in magnetic field is  $2B_s$ . If the primary input is an ac signal,  $V_1(t) = V_0 \sin(2\pi ft)$ , then Eq. (9.28) implies that

$$V_0/f \text{ (volt-s)} \leq 2\pi N_1 A_l B_s. \quad (9.29)$$

Transformers are usually run well below the limit of Eq. (9.29) to minimize hysteresis losses.

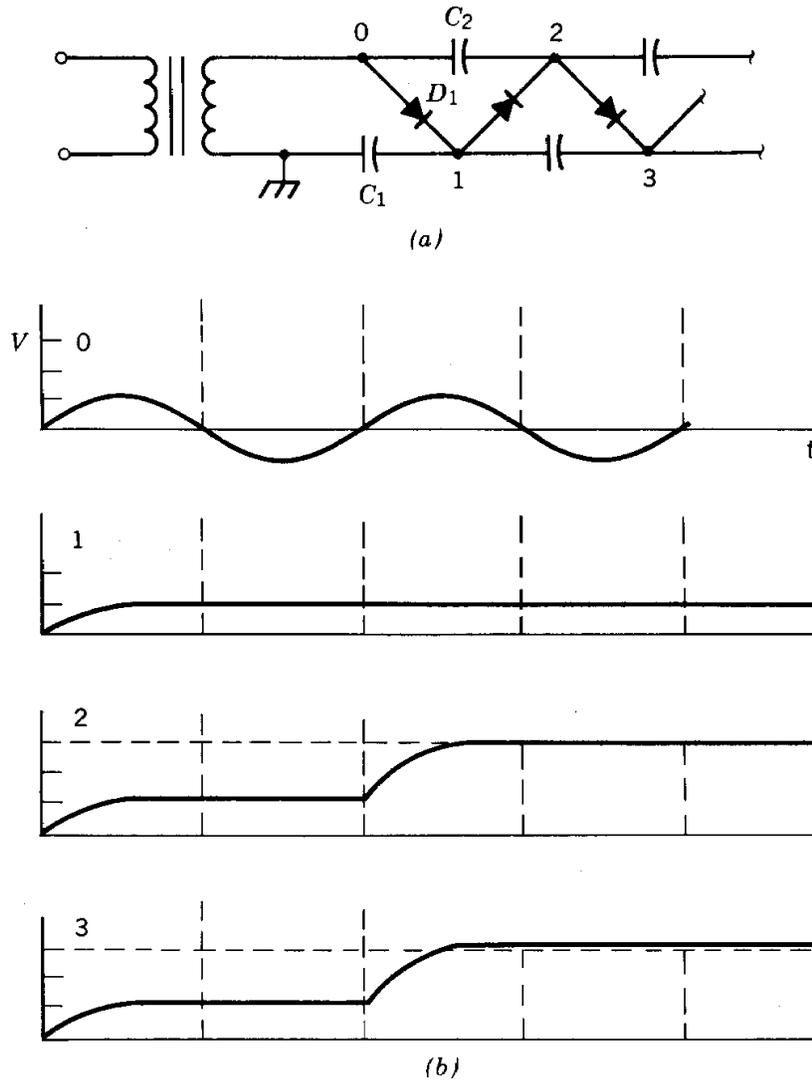
The basic circuit for a high-voltage dc supply is shown in Figure 9.9. The configuration is a *half-wave rectifier*; the diode is oriented to pass current only on the positive cycle of the transformer output. A capacitor is included to reduce ripple in the voltage. The fractional drop in voltage during the negative half-cycle is on the order  $(1/2f)/RC$ , where  $R$  is the load resistance. Output voltage is controlled by a variable autotransformer in the primary.

Because of the core volume and insulation required, transformers are inconvenient to use at voltages above 100 kV. The ladder network illustrated in Figure 9.10a can supply voltages in the 1-MV range. It is the basis of the Cockcroft-Walton accelerator [J. D. Cockcroft and E. T. S. Walton, Proc. Roy. Soc. (London) **A136**, 619 (1932)] which is utilized as a preinjector for many



**Figure 9.9** Half-wave-rectifier circuit to generate high-voltage dc power from a transformed ac waveform.

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**Figure 9.10** (a) Voltage multiplication ladder network (Cockcroft–Walton accelerator). (b) Ideal voltage waveforms (neglecting the ac generator impedance) shown at various points of the network.

high-energy accelerators. We can understand operation of the circuit by considering the voltage waveforms at the input (0) and, at the three points indicated in Figure 9.10b (1, 2, 3). The input voltage is a bipolar ac signal. In order to simplify the discussion, we assume an ideal ac voltage source that turns on instantaneously and can supply infinite current; in reality, a series resistor extends charging over many cycles to prevent damage to the transformer. On the first positive half-cycle, current flows through the diodes to charge all the points to  $+V_o$ . On the negative half-cycle, the voltage at point 1 is maintained positive because current cannot flow backward in diode  $D_1$ . The voltage at point 2 is maintained positive by conductance from  $C_1$  to  $C_2$  through diode  $D_2$ . At the time of maximum negative voltage at the input, there is a voltage difference greater than  $V_o$  across capacitor  $C_2$ . In the steady state, the voltage difference approaches  $2V_o$ . On

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the second positive cycle, the voltage at point 2 is boosted to  $3V_o$ . The voltage at point 3 also approaches  $3V_o$  because of charging through  $D_3$ . The reasoning can be extended to higher points on the ladder, leading to the unloaded steady-state voltages indicated in Figure 9.10.

At the same voltage, a ladder network requires about the same number of diodes and capacitors as a power supply based on a transformer and rectifier stack. The main advantages of a ladder network is that it utilizes a smaller transformer core and it is easier to insulate. Insulation of the secondary of a transformer is difficult at high voltage because the secondary winding must encircle the transformer core. A large core must be used with oil-impregnated insulation. In contrast, the ladder network is extended in space with natural voltage grading along the column. It is possible to operate Cockcroft-Walton-type accelerators at megavolt levels with air insulation at atmospheric pressure by locating the capacitor-diode stack in a large shielded room.

### 9.3 INSULATION

Insulation is the prevention of current flow. It is the major technological problem of high-voltage electrostatic acceleration. At low values of electric field stress, current flow through materials such as glass, polyethylene, transformer oil, or dry air is negligible. Problems arise at high voltage because there is sufficient energy to induce ionization in materials. Portions of the material can be converted from a good insulator to a conducting ionized gas (plasma). When this happens, the high-voltage supply is shorted. Plasma breakdowns can occur in the solid, liquid, or gaseous insulation of high-voltage supplies and cables. Breakdown may also occur in vacuum along the surface of solid insulators. Vacuum breakdown is discussed in the Section 9.5.

There is no simple theory of breakdown in solids and liquids. Knowledge of insulating properties is mainly empirical. These properties vary considerably with the chemical purity and geometry of the insulating material. The *dielectric strength* and relative dielectric constant of insulators commonly used in high-voltage circuits are given in Table 9.1. The dielectric strength is the maximum electric field stress before breakdown. The actual breakdown level can differ considerably from those given; therefore, it is best to leave a wide safety factor in designing high-voltage components. The dielectric strength values of Table 9.1 hold for voltage pulses of submicrosecond duration. Steady-state values are tabulated in most physical handbooks.

Measurements show that for voltage pulses in the submicrosecond range the dielectric strength of liquids can be considerably higher than the dc value. The following empirical formulas describe breakdown levels in transformer oil and purified water [see R. B. Miller, **Intense Charged Particle Beams**, (Plenum, New York, 1982), 16] two media used extensively in high-power pulse modulators. The pulsed voltage dielectric strength of transformer oil is approximately

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**TABLE 9.1 Properties of Some Common Insulators<sup>a</sup>**

Material	Relative Dielectric Constant ( $\epsilon/\epsilon_0$ )	Dielectric Strength (MV/cm)
Transformer oil	3.4	1
Mylar	3	1.8
Polyethylene	2.25	1.8
Teflon	2.1	4.3
Polycarbonate	2.96	5.5

<sup>a</sup>Adapted from J. F. Francis, *High Voltage Pulse Techniques*, Air Force Office of Scientific Research, AFOSR-74-2639-5, 1974.

$$E_{\max} \cong 0.5/t_p^{0.33} A^{0.1}. \quad (9.30)$$

The quantity  $E_{\max}$  (in megavolts per centimeter) is the highest value of electric field in the insulator at the peak of the voltage pulse. The quantity  $t_p$  is the time during which the voltage is above 63% of the maximum value (in microseconds), and  $A$  is the surface area of the high voltage electrode in centimeters squared. The breakdown level of water has the same scaling as Eq. (9.30) but is polarity dependent. The dielectric strength of water for a negative high-voltage electrode is given by

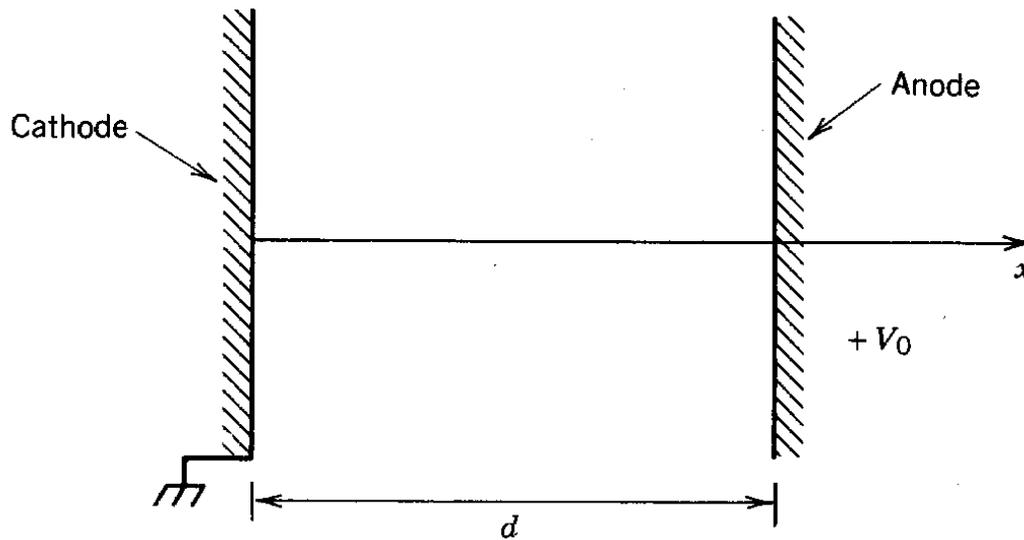
$$E_{\max} \cong 0.6/t_p^{0.33} A^{0.1} \quad (\text{negative}) \quad (9.31)$$

in the same units as Eq. (9.30). With a positive polarity, the breakdown level is approximately

$$E_{\max} \cong 0.3/t_p^{0.33} A^{0.1} \quad (\text{positive}). \quad (9.32)$$

The polarity dependence of Eqs. (9.31) and (9.32) is important for designing optimum pulsed power systems. For example, a charged coaxial transmission line filled with water can store four times the electrostatic energy density if the center conductor is negative with respect to the outer conductor rather than positive.

The enhanced dielectric strength of liquids for fast pulsed voltages can be attributed to the finite time for a breakdown streamer to propagate through the medium to short the electrodes. Section



**Figure 9.11** Gas-filled high-voltage gap, planar electrodes.

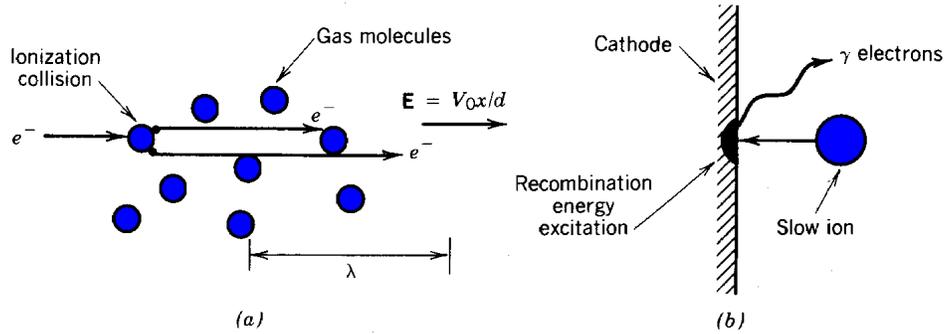
9.12 will show how pulse-charge overvoltage is used to increase the electrostatic energy density of a pulse modulator for high output power. This process can be applied in liquid insulators because they are *self-healing*. On the other hand, the damage caused by streamers is cumulative in solids. Solid insulators cannot be used above the steady-state levels.

Gas insulation is used in most steady-state high-voltage electrostatic accelerators. Gases have  $\epsilon/\epsilon_0 \sim 1$ ; therefore, gas insulators do not store a high density of electrostatic energy. Gases, like liquids, are self-healing after spark breakdowns. The major advantage of gas insulation is cleanliness. A fault, or leak in an accelerator column with oil insulation usually leads to a major cleanup operation. Although the gas itself is relatively inexpensive, the total gas insulation system is costly if the gas must be pressurized. Pressurization requires a large sealed vessel.

We shall study the theory of spark breakdown in gas in some detail. The topic is essential for the description of most types of fast high-voltage switches and it is relevant to insulation in most high-voltage electrostatic accelerators. Consider the one-dimensional gas-filled voltage gap of Figure 9.11. The electrodes have separation  $d$  and the applied voltage is  $V_0$ . There is negligible transfer of charge between the electrodes when  $V_0$  is small. We will determine how a large interelectrode current can be initiated at high voltage.

Assume that a few electrons are produced on the negative electrode. They may be generated by photoemission accompanying cosmic ray or ultraviolet bombardment. The small source current density leaving the electrode is represented by  $j_0$ . The electrons are accelerated by the applied field and move between the widely spaced gas molecules, as shown in Figure 9.12a. In a collision with a molecule, an electron is strongly deflected and much of its kinetic energy is absorbed. To construct a simple model, we assume that electrons are accelerated between molecules and lose all their directed energy in a collision. The parameter  $\lambda$  is the mean free path, the average distance between collisions. The mean free path is inversely proportional to the density of gas molecules. The density is, in turn, proportional to the pressure,  $p$ , so that  $\lambda \sim 1/p$ .

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.12** Processes in a gas-filled, high-voltage gap. (a) Electron migration in presence of a strong electric field showing electron acceleration, ionizing collision, and the mean free path. (b) Liberation of secondary electrons from a negative polarity electrode (cathode) by recombination of slow ions.

The average energy gained by an electron between collisions is

$$\Delta T = \lambda eE = \lambda eV_0/d. \quad (9.33)$$

An electron may ionize a molecule in a collision if  $\Delta T$  is high enough ( $\Delta T > 30$  eV). In the ionization process, the excess kinetic energy of the electron drives an electron out of the molecule, leaving a positive ion. The two electrons move forward under the influence of the field, producing further electrons. The current density increases geometrically along the electron drift direction. The ions drift in the opposite direction and do not contribute to ionization in the gas. The motion of drifting ions is dominated by collisions; generally, they cannot reach high enough velocity to eject electrons in a collision with a molecule.

Electron multiplication in a gas with an applied electric field is characterized by  $\alpha$ , the first Townsend coefficient [J. S. Townsend, *Electricity in Gases*, Phil. Trans. **A193**, 129 (1900)]. This parameter is defined by

$$\alpha = \frac{\text{number of ionizations induced by an electron}}{\text{cm of pathlength}}. \quad (9.34)$$

Consider an element of length  $\Delta x$  at the position  $x$ . The quantity  $n(x)$  is the density of electrons at  $x$ . According to the definition of  $\alpha$ , the total number of additional electrons produced in a volume with cross-sectional area  $dA$  and length  $\Delta x$  is

$$dN \cong \alpha n(x) dA \Delta x.$$

Dividing both sides of the equation by  $dA\Delta x$ , and taking the limit of small  $\Delta x$ , we find that

$$dn(x)/dx = \alpha n(x). \quad (9.35)$$

## Electrostatic Accelerators and Pulsed High Voltage

The solution of Eq. (9.35) implies an exponential electron density variation. Expressed in terms of the current density of electrons (assuming that the average electron drift velocity is independent of  $x$ ), Eq. (9.35) implies that

$$j(x) = j_0 \exp(\alpha x). \quad (9.36)$$

if the negative electrode is located at  $x = 0$ . The current density arriving at the positive electrode is

$$j_a = j_0 \exp(\alpha d). \quad (9.37)$$

Although the amplification factor may be high, Eq. (9.37) does not imply that there is an insulation breakdown. The current terminates if the source term is removed. A breakdown occurs when current flow is self-sustaining, or independent of the assumed properties of the source. In the breakdown mode, the current density rapidly multiplies until the voltage supply is shorted (discharged). Ion interactions at the negative electrode introduce a mechanism by which a self-sustained discharge can be maintained. Although the positive ions do not gain enough energy to ionize gas molecules during their transit, they may generate electrons at the negative electrode through secondary emission (Fig. 9.12b). This process is parametrized by the secondary emission coefficient  $\gamma$ . The coefficient is high at high ion energy ( $> 100$  keV). The large value of  $\gamma$  reflects the fact that ions have a short stopping range in solid matter and deposit their energy near the surface. Ions in a gas-filled gap have low average energy. The secondary-emission coefficient has a nonzero value for slow ions because there is available energy from recombination of the ion with an electron at the surface. The secondary emission coefficient for a zero-velocity ion is in the range  $\gamma \approx 0.02$ .

With ion interactions, the total electron current density leaving the negative electrode consists of the source term plus a contribution from secondary emission. We define the following current densities:  $j_o$  is the source,  $j_c$  is the net current density leaving the negative electrode,  $j_i$  is the ion current density arriving at the negative electrode, and  $j_a$  is the total electron flux arriving at the positive electrode. Each ionizing collision creates one electron and one ion. By conservation of charge, the ion current is given by

$$j_i = j_a - j_c, \quad (9.38)$$

Equation (9.38) states that ions and electrons leave the gap at the same rate in the steady state. The portion of the electron current density from the negative electrode associated with secondary emission is thus,

$$j_c - j_o = \gamma (j_a - j_c). \quad (9.39)$$

Equation (9.39) implies that

## Electrostatic Accelerators and Pulsed High Voltage

$$j_c = (j_0 + \gamma j_a)/(1+\gamma). \quad (9.40)$$

By the definition of  $\alpha$

$$j_a = j_c \exp(\alpha d). \quad (9.41)$$

Combining Eqs. (9.40) and (9.41) gives

$$\frac{j_a}{j_o} = \frac{\exp(\alpha d)}{1 - \gamma [\exp(\alpha d) - 1]}. \quad (9.42)$$

Comparing Eq. (9.42) with (9.37), the effects of ion feedback appear in the denominator. The current density amplification becomes infinite when the denominator equals zero. When the conditions are such that the denominator of Eq. (9.42) is zero, a small charge at the negative electrode can initiate a self-sustained discharge. The discharge current grows rapidly over a time scale on the order of the ion drift time, heating and ionizing the gas. The end result is a *spark*. A spark is a high-current-density plasma channel. The condition for spark formation is

$$\exp(\alpha d) \leq (1/\gamma) + 1 \cong 1/\gamma. \quad (9.43)$$

The discharge continues until the electrode voltage is depleted.

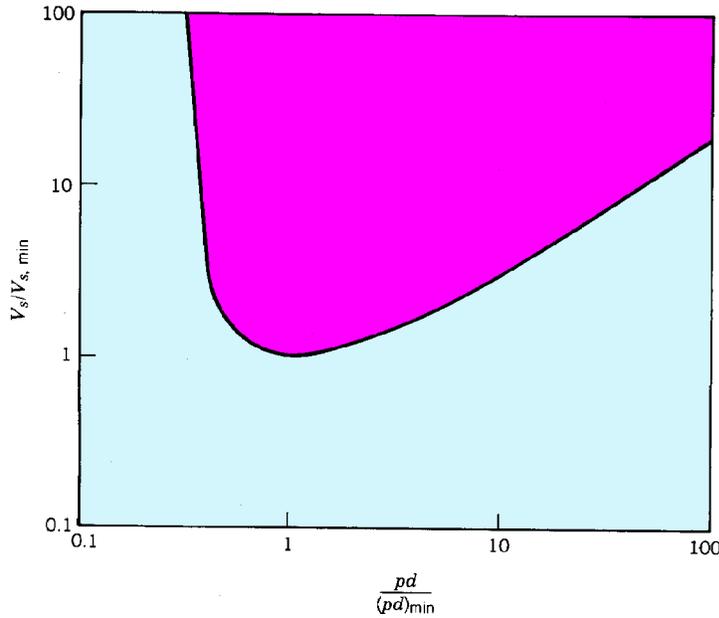
Calculation of the absolute value of  $\alpha$  as a function of the gap parameters involves complex atomic physics. Instead, we will develop a simple scaling relationship for  $\alpha$  as a function of gas pressure and electric field. Such a relationship helps to organize experimental data and predict breakdown properties in parameter regimes where data is unavailable. The ionization coefficient is proportional to the number of collisions between an electron and gas atoms per centimeter traveled, or  $\alpha \sim 1/\lambda$ . The ionization coefficient also depends on the average electron drift energy,  $E\lambda$ . The ionization rate is proportional to the fraction of collisions in which an electron enters with kinetic energy greater than the ionization energy of the atom,  $I$ . The problem of a particle traveling through a random distribution of collision centers is treated in texts on nuclear physics. The familiar result is that the distance traveled between collisions is a random variable that follows a Poisson distribution, or

$$P(x) = \exp(-x/\lambda), \quad (9.44)$$

where  $\lambda$  is the mean free path. The energy of a colliding electron is, by definition,  $T = xeV_0/d$ . The fraction of collisions in which an electron has  $T > I$  is given by the integral of Eq. (9.44) from  $x = Id/eV_0$  to  $\infty$ .

The result of these considerations is that  $\alpha$  is well described by the scaling law

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**Figure 9.13** Normalized Paschen curve; sparking voltage versus the product of pressure and gap width.

$$\alpha = (A'/\lambda) \exp(-B'd/V_o\lambda) = (Ap) \exp(-Bpd/V_o). \quad (9.45)$$

The quantities  $A$  and  $B$  are determined from experiments or detailed collision theory. The gap voltage for sparking is found by substituting Eq. (9.45) into (9.43):

$$\ln(1/\gamma) = (Apd) \exp(-Bpd/V_s).$$

Solving for  $V_s$ ,

$$V_s = Bpd / \ln\left(\frac{Apd}{\ln(1/\gamma)}\right). \quad (9.46)$$

Equation (9.46) is Paschen's law [F. Paschen, Wied. Ann. **37**, 69 (1889)] for gas breakdown. The sparking voltage is a function of the product  $pd$  and constants that depend on the gas properties. The values of  $A$  and  $B$  are relatively constant over a wide voltage range when the average electron energy is less than  $I$ . Furthermore,  $V_s$  is insensitive to  $\gamma$ ; therefore, the results are almost independent of the electrode material.

Figure 9.13 is a normalized plot of  $V_s$  versus  $pd$ . The sparking voltage reaches a minimum value

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**TABLE 9.2 Minimum Sparking Constants<sup>a</sup>**

Gas	$V_{s,\min}$ (V)	$(pd)_{\min}$ (torr-cm)
Air (dry)	327	0.567
A	137	0.9
H <sub>2</sub>	273	1.15
He	156	4.0
CO <sub>2</sub>	420	0.51
N <sub>2</sub>	251	0.67
O <sub>2</sub>	450	0.7

<sup>a</sup>Adapted from J. D. Cobine, *Gaseous Conductors*, Dover, New York, 1958, p. 164.

$V_{s,\min}$  at a value  $(pd)_{\min}$ . Voltage hold-off increases at both low and high values of  $pd$ . Voltage hold-off is high at low  $pd$  because there is a small probability that an electron will strike a molecule while traveling between electrodes. Paschen's law does not hold at very low vacuum because the assumption of a Poisson distribution of mean free paths is invalid.

At high values of  $pd$ ,  $V_s \sim (pd)$ . In this regime, electrons undergo many collisions, but the mean free path is short. Few electrons gain enough energy to produce an ionization.

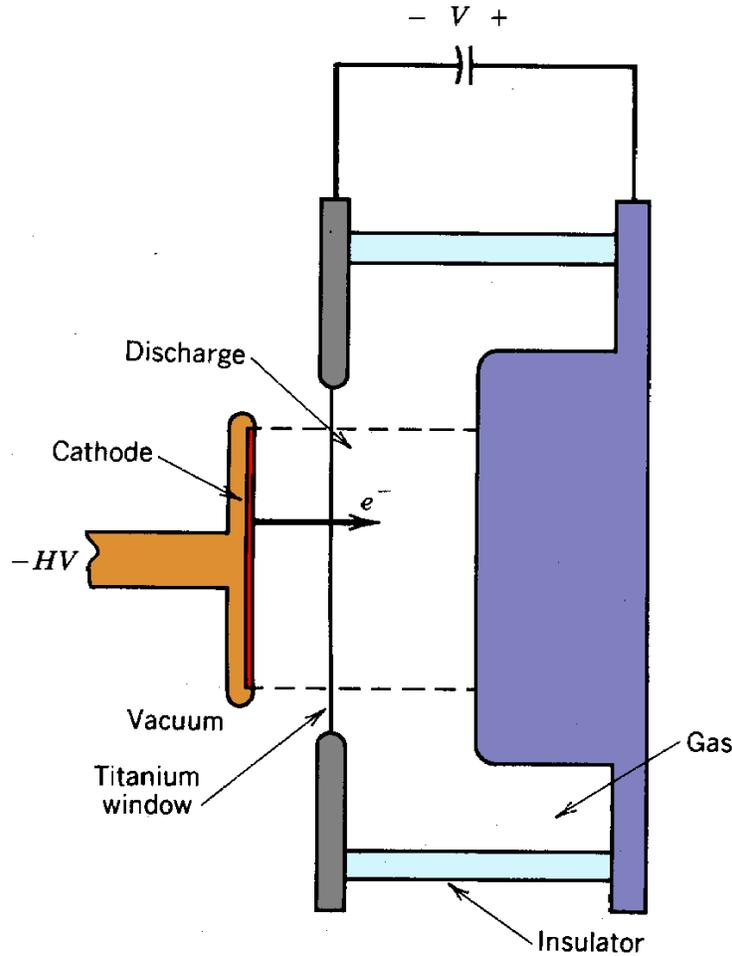
Equation (9.46) can be rewritten,

$$V_s = \frac{V_{s,\min} [pd/(pd)_{\min}]}{\ln[2.72 pd/(pd)_{\min}]} \quad (9.47)$$

Table 9.2 gives a table of spark parameters for some common gases. Note that the minimum sparking voltage is high for electronegative gases like oxygen and low for gases with little probability of electron capture like helium and argon. In electronegative gases, there is a high probability that electrons are captured to form negative ions. The heavy negative ions cannot produce further ionization, so that the electron is removed from the current multiplication process. The result is that gaps with electronegative-gases can sustain high voltage without breakdown. Sulfur hexafluoride an extremely electronegative gas, is often mixed with air or used alone to provide strong gas insulation at high pressure. The expense of the gas is offset by cost savings in the pressure vessel surrounding the high-voltage system. The addition of 8% (by volume) SF<sub>6</sub> to air increases  $V_s$  by a factor of 1.7. Pure SF<sub>6</sub> has  $V_s$  2.2 times that of air for the same  $pd$ . Vacuum insulation is seldom used in power supplies and high-voltage generators because of technological difficulties in maintaining high vacuum and the possibility of long breakdown paths. Vacuum insulation in accelerator columns is discussed in Section 9.5.

Our discussion of gas breakdown has been limited to a one-dimensional geometry. Two-dimensional geometries are difficult to treat analytically because a variety of electron trajectories are possible and the electric field varies along the electron paths. Breakdown voltages,

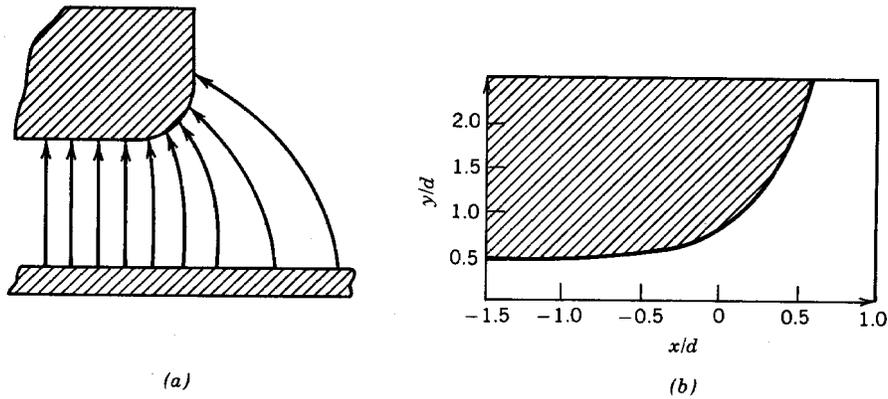
## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.14** Electron-beam-controlled gas discharge laser.

are tabulated for special electrode geometries such as two spheres. In some circumstances (such as gas lasers), a stable, uniform discharge must be sustained over a large area, as shown in Figure 9.14. The interelectrode voltage must be less than  $V_s$  to prevent a localized spark; therefore, a source current is required to sustain the discharge. The source is often provided by injections of a high-energy electron beam through a foil on the negative electrode. Demands on the source are minimized if there is a large multiplication factor,  $\exp(\alpha d)$ ; therefore, the main gap is operated as close to  $V_s$  as possible. In this case, care must be taken with shaping of the electrodes. If the electrode has a simple radius (Fig. 9.15a), then the field stress is higher at the edges than in the body of the discharge, leading to sparks. The problem is solved by special shaping of the edges. One possibility is the Rogowski profile [W. Rogowski and H. Rengier, *Arch. Elekt.* **16**, 73 (1926)] illustrated in Figure 9.15b. This shape, derived by conformal mapping, has the property that the field stress on the electrodes decreases monotonically moving away from the axis of symmetry. The shape for two symmetric profiled electrodes is described by the parametric equations

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**Figure 9.15** Electrode shaping. (a) Field enhancement at the edge of an electrode with a simple radius. (b) Electrode with a Rogowski profile.

$$\begin{aligned} x &= d\varphi/\pi, \\ y &= (d/\pi) [\pi/2 + \exp(\varphi)]. \end{aligned} \tag{9.48}$$

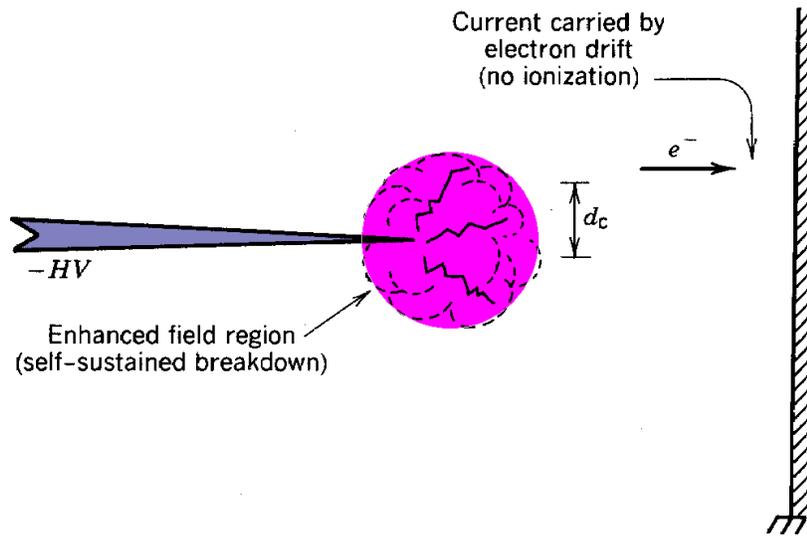
where  $d$  is the minimum separation between electrodes. As indicated in Figure 9.15b,  $x$  is the transverse distance from the center of the electrodes, and  $\pm y$  is the distance from the midplane (between electrodes) to the electrode surfaces.

*Corona discharges* appear in gases when electrodes have strong two-dimensional variations (Figure 9.16). Corona (crown in Latin) is a pattern of bright sparks near a pointed electrode. In such a region, the electric field is enhanced above the breakdown limit so that spark discharges occur. Taking the dimension of the corona region as  $d$ , the approximate condition for breakdown is

$$\bar{E}d_c \approx V_{s,\min} (pd_c/(pd)_{\min}) / \ln[2.72(pd_c/(pd)_{\min})] \tag{9.49}$$

where  $\bar{E}$  is the average electric field in the corona region. A self-sustained breakdown cannot be maintained by the low electric field in the bulk of the gap. Current in the low-field region is conducted by a Townsend (or dark) discharge, with the corona providing the source current  $j_o$ . The system is stabilized by the high resistivity of the dark discharge. A relatively constant current flows, even though the sparks of the corona fluctuate, extinguish, and reform rapidly. Inspection of Eq. (9.49) shows that for constant geometry, the size of the corona region grows with increasing electrode voltage. The voltage drop in the highly ionized corona is low; therefore, the voltage drop is concentrated in the dark discharge region. At some voltage, bulk breakdown occurs and the electrodes are shorted.

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.16** Physical basis for corona discharge near a pointed electrode.

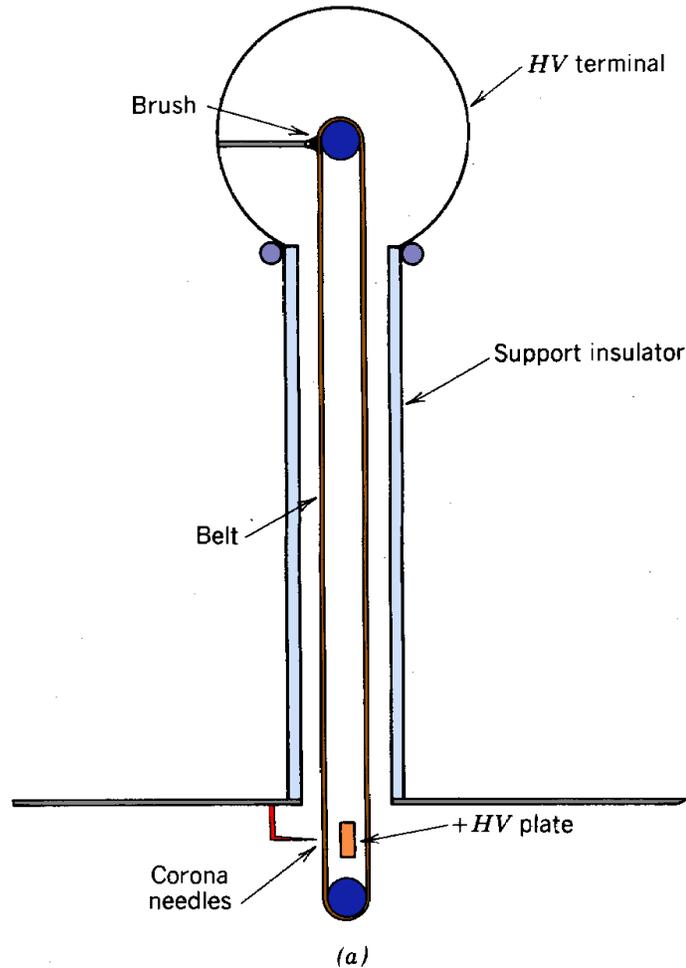
### 9.4 VAN DE GRAAFF ACCELERATOR

Two types of voltage generators are used for low current electrostatic accelerators in the megavolt range, the Cockcroft-Walton and Van de Graaff generators. We have already discussed the principle of the ladder network voltage used in the Cockcroft-Walton accelerator in Section 9.2. Cockcroft-Walton accelerators are used mainly for injectors with voltage of approximately 1 MV. In this section, we shall discuss the Van de Graaff generator, which can sustain steady-state voltages up to 15 MV.

A Van de Graaf [R. J. Van de Graaff, *Phys. Rev.* **38**, 1919 (1931)] accelerator for electrons is illustrated in Figure 9.17. The principle of operation is easily understood. A corona discharge from an array of needles in gas is used as a convenient source of electrons. The electrons drift toward the positive electrode and are deposited on a moving belt. The belt composed of an insulating material with high dielectric strength, is immersed in insulating gas at high pressure. The attached charge is carried mechanically against the potential gradient into a high-voltage metal terminal. The terminal acts as a Faraday cage; there is no electric field inside the terminal other than that from the charge on the belt. The charge flows off the belt when it comes in contact with a metal brush and is deposited on the terminal.

The energy to charge the high-voltage terminal is supplied by the belt motor. The current available to drive a load (such as an accelerated beam) is controlled by either the corona discharge current or the belt speed. Typical currents are in the range of  $10 \mu\text{A}$ . Power for ion sources or thermionic electron sources at high voltage can be supplied by a generator attached to the belt pulley inside the high-voltage terminal. The horizontal support of long belts and accelerator

## Electrostatic Accelerators and Pulsed High Voltage

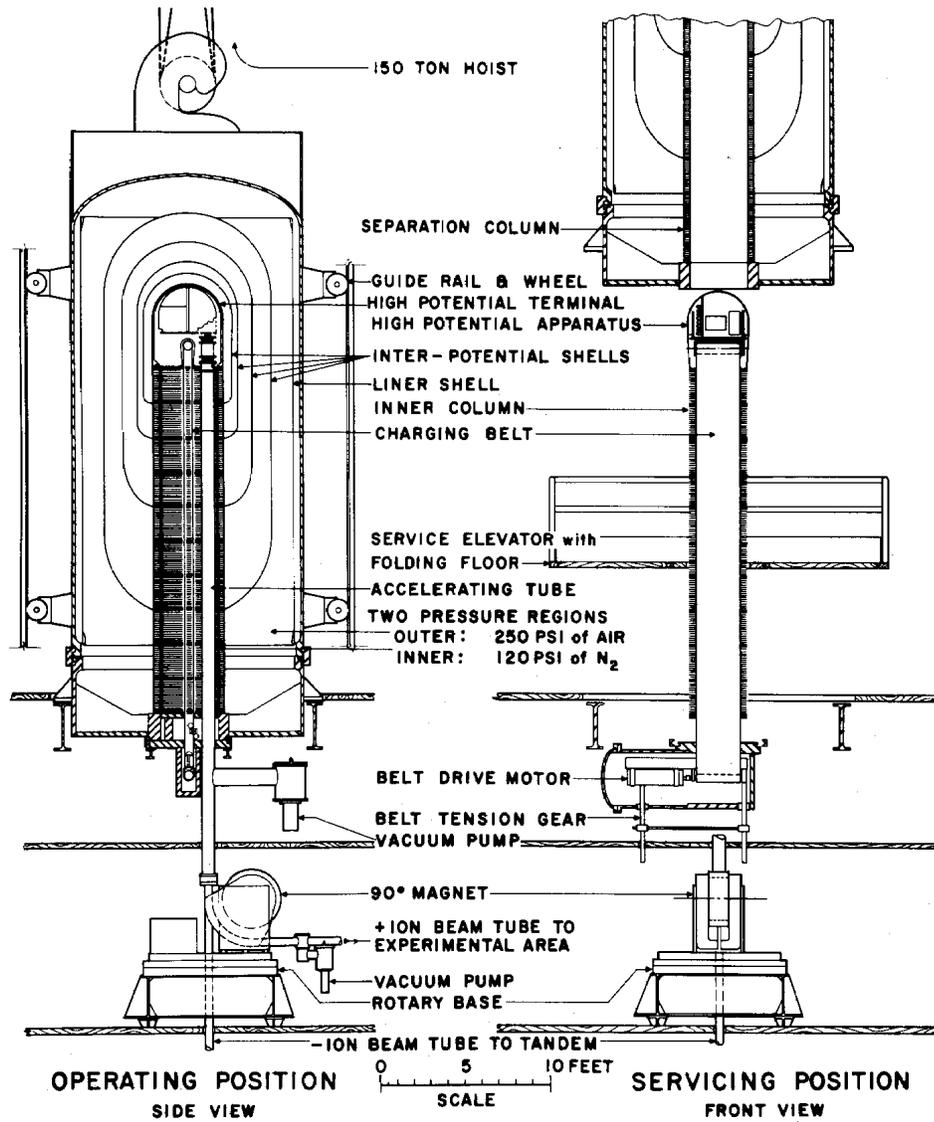


**Figure 9.17** Van de Graaff accelerator. (a) Principle of operation. (b) Los Alamos National Laboratory, 7 MeV Van de Graaff accelerator, utilized as injector for 24.5 MeV tandem Van de Graaff accelerator facility. (Courtesy, J. R. Tesmer, Los Alamos National Laboratory.)

columns is difficult; therefore, many high-voltage Van de Graaff accelerators are constructed vertically, as in Figure 9.17.

Van de Graaff accelerators are excellent research tools because they provide a steady-state beam with good energy regulation. Continuous low-current beams are well suited to standard nuclear diagnostics that detect individual reaction products. Although the primary use of Van de Graaff accelerators has been in low-energy nuclear physics, they are finding increased use for high-energy electron microscopes and ion microprobes. The output beam energy of a Van de Graaff accelerator can be extended a factor of 2 (up to the 30 MeV range) through the tandem configuration illustrated in Figure 9.19. Negative ions produced by a source at ground potential are accelerated to a positive high-voltage terminal and pass through a stripping cell. Collisions in the cell remove electrons, and some of the initial negative ions are converted to positive ions. They are further

## Electrostatic Accelerators and Pulsed High Voltage



### LOS ALAMOS VERTICAL VAN de GRAAFF

Figure 9.17 (Continued).

accelerated traveling from the high-voltage terminal back to ground.

Voltage hold-off is maximized when there are no regions of enhanced electric field. In other words, the electric field stress should be made as uniform as possible in the region of gas insulation. High-voltage terminals are usually constructed as large, smooth spheres to minimize peak electric field stress. We can show that, given the geometry of the surrounding grounded pressure vessel, there is an optimum size for the high-voltage terminal. Consider the geometry of Figure 9.19a. The high-voltage terminal is a sphere of radius  $R_1$  inside a grounded sphere of radius  $R_2$ . Perturbing effects of the belt and accelerator column are not included. The solution to

## Electrostatic Accelerators and Pulsed High Voltage

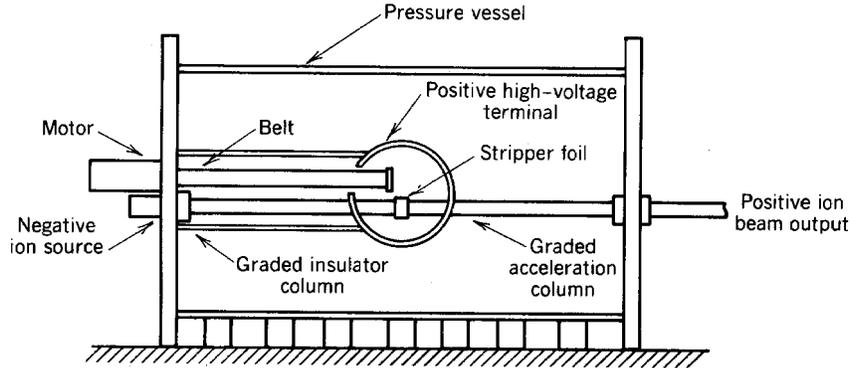


Figure 9.18 Schematic diagram of a tandem Van de Graaff accelerator.

the Laplace equation in spherical coordinates gives the radial variation of potential between the spheres

$$\varphi(r) = \frac{V_0 R_0 (R_2/r - 1)}{R_2 - R_0} . \quad (9.50)$$

Equation (9.50) satisfies the boundary conditions  $\varphi(R_0) = V_0$  and  $\varphi(R_2) = 0$ . The radial electric field is

$$E_r(r) = -\partial\varphi/\partial r = \frac{V_0 R_0 (R_2/r^2)}{R_2 - R_0} . \quad (9.51)$$

The electric field is maximum on the inner sphere,

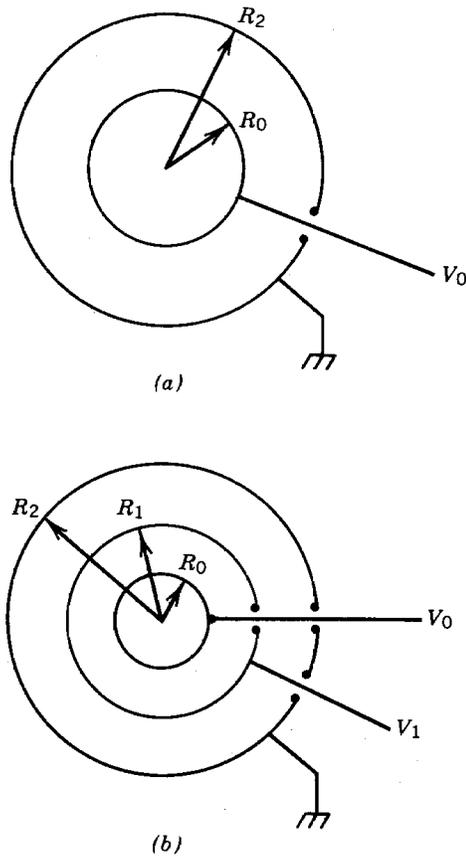
$$E_{r,\max} = \frac{(V_0/R_2) (R_2/R_0)}{1 - R_0/R_2} . \quad (9.52)$$

We assume  $R_2$  is a fixed quantity, and determine the value of  $R_0/R_2$  that will minimize the peak electric field. Setting  $\partial E_{r,\max}/\partial(R_0/R_2) = 0$ , we find that

$$R_0/R_2 = 1/2. \quad (9.53)$$

A similar calculation can be carried out for concentric cylinders. In this case, the optimum, ratio of inner to outer cylinder radii is

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.19** Geometry for calculating electric fields on spherical electrodes. (a) Two concentric spheres. (b) Concentric spheres with an equipotential shield.

$$R_0/R_2 = 1/e = 0.368. \quad (9.54)$$

A better electric field distribution can be obtained through the use of equipotential shields. Equipotential shields are biased electrodes located between the high-voltage terminal and ground, as in Figure 9.17. Voltage on the shields is maintained at specific intermediate values by a high-voltage resistive divider circuit. The simplified geometry of three nested spheres (Fig. 9.19b) will help in understanding the principle of equipotential shields. The quantities  $R_2$  and  $V_0$  are constrained by the space available and the desired operating voltage. We are free to choose  $R_0$ ,  $R_1$ , and  $V_1$  (the potential of the shield) for optimum voltage hold-off. The electric fields on outer surfaces of the nested spheres,  $E_1(r=R_1^+)$  and  $E_0(r=R_0^+)$  are simultaneously minimized.

The fields on the outer surfaces of the electrodes are

## Electrostatic Accelerators and Pulsed High Voltage

$$E_0 = \frac{(V_0 - V_1)/R_0}{1 - R_0/R_1}, \quad (9.55)$$

$$E_1 = \frac{V - 1/R_1}{1 - R_1/R_1}. \quad (9.56)$$

The following equations are satisfied when the surface fields are minimized:

$$\Delta E_0 = (\partial E_0 / \partial V_1) \Delta V_1 + (\partial E_0 / \partial R_1) \Delta R_1 + (\partial E_0 / \partial R_0) \Delta R_0 = 0, \quad (9.57)$$

$$\Delta E_1 = (\partial E_1 / \partial V_1) \Delta V_1 + (\partial E_1 / \partial R_1) \Delta R_1 + (\partial E_1 / \partial R_0) \Delta R_0 = 0, \quad (9.58)$$

The final term in Eq. (9.58) is zero since there is no explicit dependence of  $E_1$  on  $R_0$ . The previous study of two nested spheres implies that  $E_0$  is minimized with the choice  $R_0 = R_1/2$  given  $V_1$  and  $R_1$ . In this case, the last term in Eq. (9.57) is independently equal to zero. In order to solve the reduced Eqs. (9.57) and (9.58), an additional equation is necessary to specify the relationship between  $\Delta R_1$  and  $\Delta V_1$ . Such a relationship can be determined from Eqs. (9.55) and (9.56) with the choice  $E_0 = E_1$ . After evaluating the derivatives and substituting into Eqs. (9.57) and (9.58), the result is two simultaneous equations for  $R_1$  and  $V_1$ . After considerable algebra, the optimum parameters are found to be

$$R_0 = R_1/2 \quad R_1 = 5R_2/8, \quad V_1 = 3V_0/5. \quad (9.59)$$

Addition of a third electrode lowers the maximum electric field stress in the system. To compare, the optimized two electrode solution (with  $R_0 = R_2/2$ ) has

$$E_0 = 4V_0/R_2. \quad (9.60)$$

The peak field for the three electrode case is

$$E_0 = E_1 = (192/75) V_0/R_2 = 2.56 V_0/R_2. \quad (9.61)$$

Larger numbers of nested shells reduce the peak field further. The intent is to counter the geometric variation of fields in spherical and cylindrical geometries to provide more uniform voltage grading. For a given terminal and pressure vessel size, multiple shells have approximately equal voltage increments if they are uniformly spaced.

## 9.5 VACUUM BREAKDOWN

A column for electron beam acceleration in an electrostatic accelerator is shown in Figure 7.14. It consists of insulating disks separated by metal grading rings. The disks and rings are sealed to hold high vacuum, either by low vapor pressure epoxy or by a direct metal to ceramic bond. We have already discussed the role grading rings play in focusing particle beams through long columns (Section 7.7). We shall now consider how rings also improve the voltage hold-off capability of solid insulators in vacuum. High-vacuum sparking is determined by complex phenomena occurring on solid surfaces under high-field stress. Effects on both conductors and insulators can contribute to breakdown.

When a metal is exposed to a strong electric field, electrons may be produced by field emission. Field emission is a quantum mechanical tunneling process. Strong electric fields lower the energy barrier at the metal surface, resulting in electron emission. Field emission is described by the Fowler-Nordheim equation [see J. Thewlis (Ed.), **Encyclopaedic Dictionary of Physics, Vol. 3**, (Macmillan, New York, 1962), 120]. which has the scaling

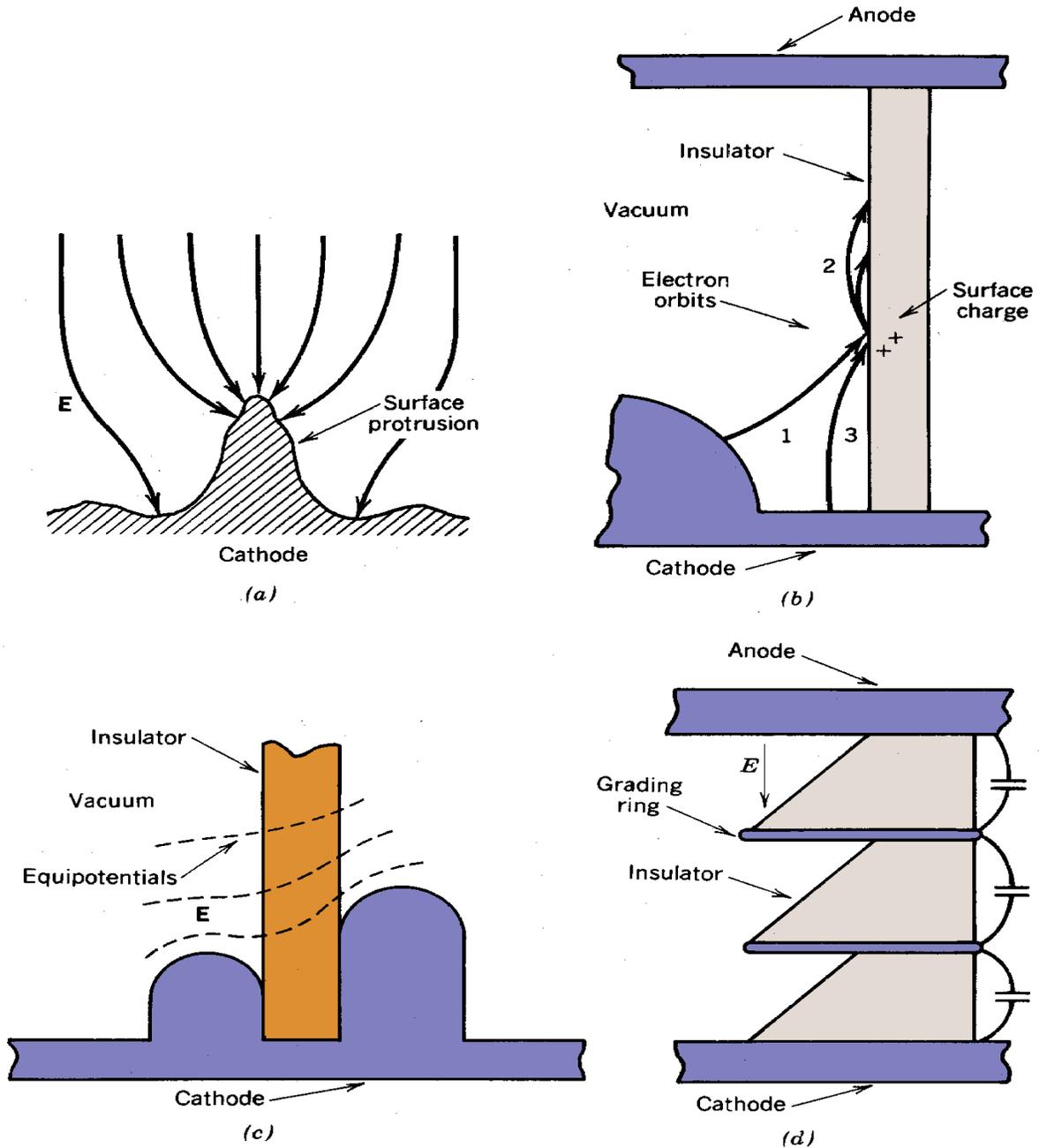
$$j_{ef} \sim E^2 \exp(-B/E), \quad (9.62)$$

where  $E$  is the electric field. Typical current density predicted by Eq. (9.62) in a gap, with an electric field of 25 MV is on the order of 10 nA/cm<sup>2</sup>.

Equation (9.62) implies that a plot of  $\ln(j_{ef}/E^2)$  versus  $1/E$  is a straight line. The scaling is observed in experiments on clean metal surfaces, but the slope of the plot indicates that the field magnitude on the surface is 10-100 times the macroscopic field (voltage divided by gap spacing). The discrepancy is explained by the presence of whiskers on the metal surface [see P. A. Chatterton, *Vacuum Breakdown*, in **Electrical Breakdown in Gases**, edited by J. M. Meek and J. D. Craggs (Wiley, New York, 1978)]. Whiskers are small-scale protrusions found on all ordinary metal surfaces. The electric field is enhanced near whiskers, as shown in Figure 9.20a. Although the high-field regions account for a small fraction of the surface area, there is a significant enhancement of emitted current because of the strong scaling of Eq. (9.62) with  $E$ .

The area-averaged whisker-enhanced current density from a metal surface at 10 MV/m is only 1-100  $\mu\text{A}/\text{cm}^2$ . The leakage current by this process is not a significant concern in high-voltage systems. The main problem is that whiskers may be vaporized by the high-current density at the tips, ejecting bursts of material into highly stressed vacuum regions. Vaporization of even a small amount of solid material (at a density of  $10^{22} \text{ cm}^{-3}$ ) on the tip of the whisker can eject a significant gas pulse into the vacuum. A conventional gas spark can then occur. Vaporization occurs at field stress exceeding 10 MV/m (100 kV/cm). kV/cm) for machined metal surfaces. The level can be increased by careful chemical preparation of the surface or by conditioning. In conditioning, a

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.20** Processes influencing electric breakdowns in vacuum. (a) Microscopic view of surface of negative electrode showing field enhancement on tips of whiskers. (b) High-voltage electrodes separated by insulator; processes leading to insulator breakdown: (1) Electrons emitted from cathode (by field emission or whisker vaporization) collide with insulator surface. (2) If secondary emission coefficient exceeds unity, electrons are liberated. (3) Excess positive charge attracts other electrodes, increasing growth of surface charge. (c) Shielding triple point to prevent deposition of electrons emitted from negative electrode on insulator. (d) Common high-voltage insulator design, with insulator shaping (to minimize charge imbalance on surfaces) and grading rings.

## Electrostatic Accelerators and Pulsed High Voltage

metal surface is slowly raised to the final electric field operating level. Controlled vaporization of whiskers often smooths the surface, allowing higher operating field.

Whisker vaporization probably accounts for *microbursts* in steady-state electrostatic accelerators. Microbursts are random pulses of current observed in highly stressed accelerator columns. Whisker vaporization is used to advantage in pulsed electron accelerators. In this application, a rapidly pulsed electric field is applied to the cathode surface. Many whiskers vaporize simultaneously, covering the cathode with a dense, cold plasma. The plasma has zero work function; electrons can be extracted at current density exceeding  $1 \text{ kA/cm}^2$ . The main problem with *cold cathodes* is that the plasma expands into the acceleration gap (*plasma closure*), causing deterioration of the beam optics and ultimately shorting the gap. The expansion rate of the plasma, the *closure velocity*, is enhanced by electron flow through the plasma. Beam plasma instabilities rapidly beat the plasma. Closure velocities as high as  $10 \text{ cm}/\mu\text{s}$  have been observed. Depending on the acceleration gap width, cold cathodes are useful only for pulses in the  $1\text{-}\mu\text{s}$  range.

Breakdowns occur on insulator surfaces in a vacuum at lower field levels than those that cause whisker vaporization on metals. Steady-state operating levels for insulators are in the  $2 \text{ MV/m}$  ( $20 \text{ kV/cm}$ ) range. Therefore, vacuum insulators are the weak point in any high-voltage electrostatic accelerator. The mechanisms of breakdown are not well understood. The following qualitative observations lead to useful procedures for optimizing insulator hold-off

1. There are two main differences between insulators and metals that affect vacuum breakdown. First, electric fields parallel to surfaces can exist near insulators. Second, regions of space charge can build up on insulators. These charges produce local distortions of electric fields.
2. A full-scale breakdown on an insulator is a complex process. Fortunately, it is usually sufficient to understand the low-current initiation processes that precede breakdown to predict failure levels.
3. Discharges on insulators are not initiated by whisker explosion because current cannot flow through the material. If the electric field is below the bulk dielectric strength of the insulator, discharge initiation is probably caused by processes on nearby metal surfaces.
4. When voltage pulses are short ( $< 50 \text{ ns}$ ), current flow from field emission on nearby electrodes is too small to cause serious charge accumulation on an insulator. In this case, the onset of insulator breakdown is probably associated with charge from whisker explosions on the metal electrodes. This is consistent with experiments; short-pulse breakdown levels on insulators between metal plates are in the  $10\text{-}15 \text{ MV/m}$  range.
5. Electrons produced in a whisker explosion move away from a metal electrode because the electric field is normal to the surface. In contrast, electrons can be accelerated parallel to the surface of an insulator. High-energy electrons striking the insulator can lead to

## Electrostatic Accelerators and Pulsed High Voltage

ejection of surface material. A microburst between metal electrodes in vacuum has a slow growth of discharge current (over many ion transit times) and may actually quench. In contrast, a discharge initiated along an insulator has a rapid growth of current and usually leads to a complete system short.

6. For long voltage pulses, field emission from nearby metal electrodes leads to accumulated space charge on insulator surfaces and field distortion (Fig.9.20b). Secondary emission coefficients on insulators are generally above unity in the energy range from 100 eV to a few keV. The impact of field-emitted electrons results in a net positive surface charge. This charge attracts more electrons, so that the electric field distortion increases. The nonuniform surface electric field fosters breakdown at levels well below that for whisker explosion on surrounding electrodes ( $\sim 2$  MV/m).

There are some steps that can be taken to maximize voltage hold-off along an insulator surface:

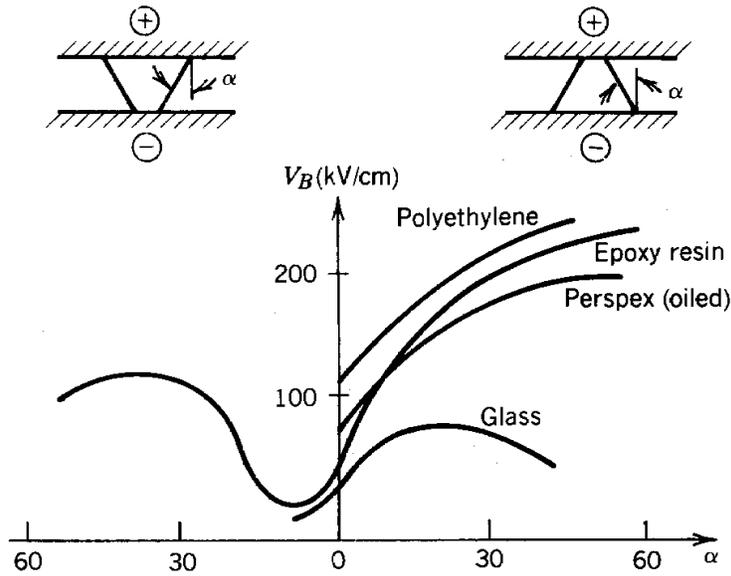
1. The *triple point* is the location where metal and insulator surfaces meet in a vacuum. In high-voltage pulsed systems, whisker explosions near the triple points at both ends of the insulator can spray charge on the insulator surface. Electrostatic shielding of the triple point to prevent whisker explosions, as shown in Figure 9.20c, improves voltage hold-off.

2. Electrons from whisker explosions are the most dangerous particles because they travel rapidly and have a secondary emission coefficient about two orders of magnitude higher than ions in the keV range. Voltage hold-off is high if the insulator is shaped so that electrons emitted from the negative electrode travel directly to the positive electrode without striking the insulator surface. A common insulator configuration is illustrated in Figure 9.20d. Insulator sections with angled surfaces are separated by grading rings. With the angle shown, all electrons emitted from either metal or insulator surfaces are collected on the rings. The hold-off level in both pulsed and dc columns may be doubled with the proper choice of insulator angle. A graph of breakdown level for pulsed voltages as a function of insulator angle is shown in Figure 9.21.

3. Space-charge-induced field distortions in dc columns are minimized if the insulator is divided into a number of short sections separated by metal grading rings. The potential drop between rings is evenly distributed by a divider network outside the vacuum. Current flowing through the divider relieves charge accumulate on the rings from field emission currents.

4. Many pulsed voltage insulating columns are constructed with grading rings in the configuration shown in Figure 9.20d., although the utility of rings is uncertain. Capacitive grading is usually sufficient to prevent field distortion for fast pulses, especially if the medium outside the vacuum insulator has a high dielectric constant. Vacuum insulators in

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.21** Insulator breakdown levels as function of insulator inclination angle for materials subject to fast voltage pulse ( $\sim 30$  ns). (Adapted from I. D. Smith, *Proc. First Int. Symp. Discharges and Elec. Insul. in Vac.*, MIT, Cambridge MA, 1964, p. 261.)

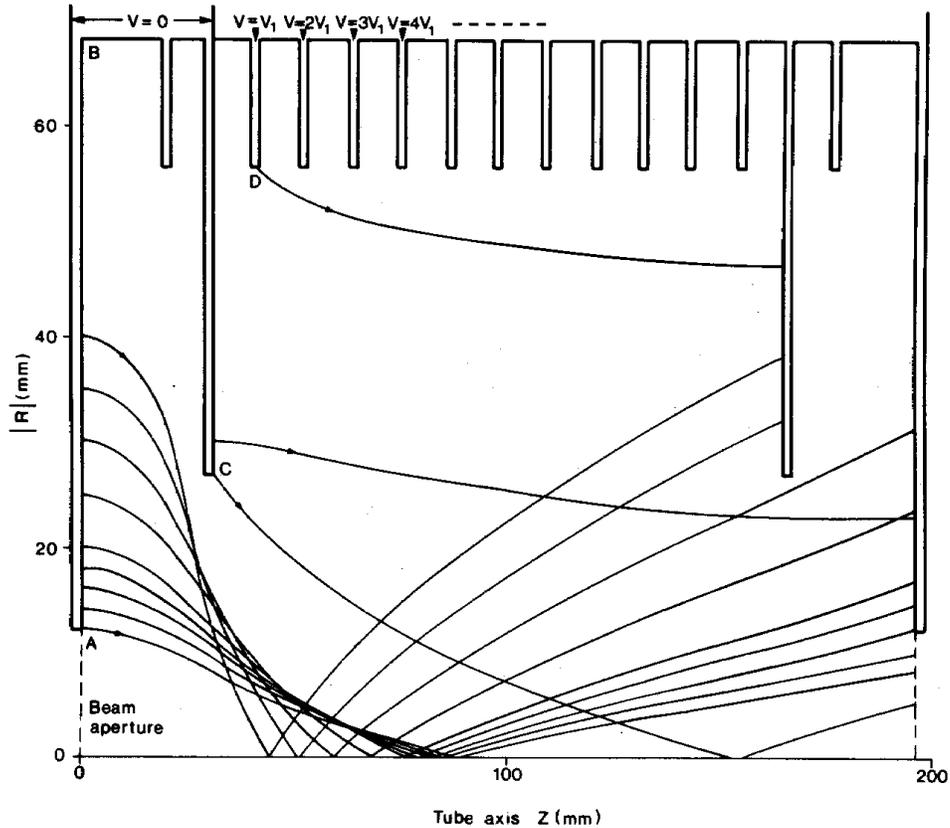
pulsed voltage systems usually have a breakdown level proportional to their length along the field.

Insulator hold-off in dc accelerator columns may be reduced with the introduction of a beam. Peripheral beam particles may strike the insulator directly or the beam may produce secondary particles. Beam induced ions are dangerous if they reach energy in the range  $> 100$  keV. These ions have a high coefficient of secondary emission ( $> 10$ ) and deposit large energy in a narrow layer near the surface when they collide with an insulator. Figure 9.22 illustrates a section of a Van de Graaff accelerator column designed to minimize insulator bombardment. Grading rings are closely spaced and extend inward a good distance for shielding. Furthermore, the gradient along the rings is purposely varied. As we saw in studying the paraxial ray equation (Section 7.5), variation of  $E_z$  has an electrostatic lens effect on low-energy particle orbits. Secondary electrons, ions, and negative ions are overfocused and collected on the rings before they are able to accelerate to high energy along the column.

## 9.6 LRC CIRCUITS

This section initiates our study of pulsed voltage generators. All pulsed voltage circuits have an energy storage element where electrical energy is contained in the form of electric or magnetic fields. The energy is transferred by a fast switch to a load. The speed of transfer (or the maximum

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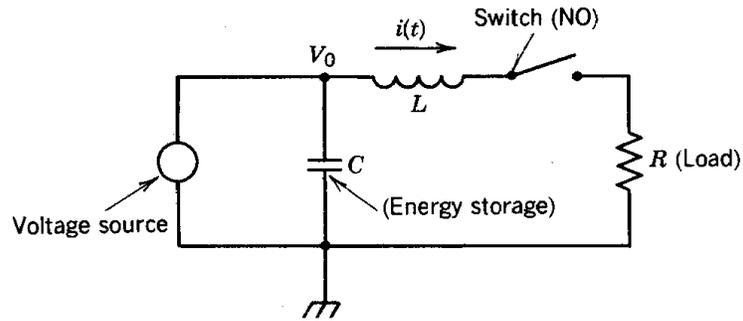
**Figure 9.22** Insulator column for a Van de Graaff accelerator showing orbits of secondary ions liberated from electrodes. Variation of the grading ring spacing results in electrostatic overfocusing and radial loss of ions. (Courtesy T. Joy, Daresbury Laboratory.)

power attainable) is limited by parasitic inductance or capacitance in the circuit. The voltage pulse waveform is determined by the configuration of the energy storage element and the nature of the load. In the next sections we shall concentrate on simple resistive loads. The combination of energy storage element and switch is usually called a *voltage modulator*. The circuit produces a modulation, or variation in time, of the voltage.

There are four main accelerator applications of pulsed voltage circuits.

1. High electric field can often be sustained in small systems for short times because of time-dependent processes controlling breakdowns. Sparking on vacuum insulators is one example. Therefore, pulsed voltage modulators can be used to generate rapidly pulsed high-energy beams in compact systems.
2. Pulsed accelerators are often required for the study of fast physical processes. Applications include pulsed X-ray radiography, material response at high pressure and temperatures, and inertial fusion.

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.23** Pulse modulator with capacitive energy storage and closing switch.

3. Some beam applications require power at the multimegawatt level. Such systems are usually run at low-duty cycle. A system capable of supplying such power on a steady-state basis would require extensive ancillary power and cooling equipment. In contrast, a pulsed power modulator stores energy over a long time and releases it in a short pulse. This process is called *power compression*. High-energy electron linacs (Section 14.1) illustrate this process. Strong accelerating gradient is obtained by injecting pulsed electromagnetic energy into a slow-wave structure. Klystron tubes powered by modulated dc generate the rf power.

4. Accelerators that utilize inductive isolation by ferromagnetic cores, such as the induction linac and the betatron, must operate in a pulsed mode.

The simplest electrical energy storage device is a single capacitor. The voltage modulator of Figure 9.23 consists of a capacitor (charged to voltage  $V_0$ ) and a shorting switch to transfer the energy. The energy is deposited in a load resistor,  $R$ . The flow of current involved in the transfer generates magnetic fields, so we must include the effect of a series inductance  $L$  in the circuit.

The time-dependent voltage across the circuit elements is related to the current by

$$V_C = -V_0 = \int i dt/C,$$

$$V_L = L di/dt.$$

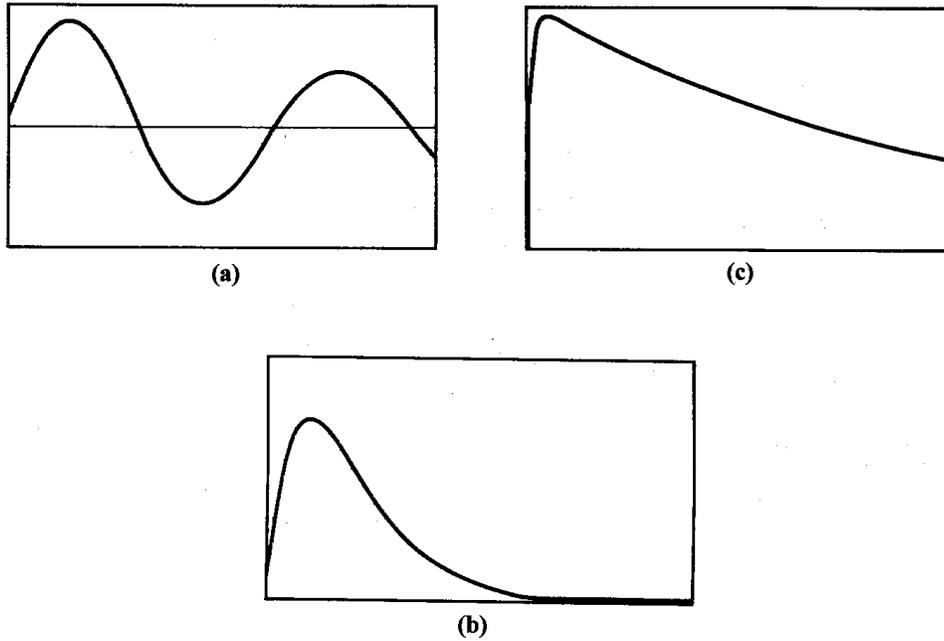
$$V_R = iR.$$

Setting the loop voltage equal to zero gives

$$L (d^2i/dt^2) + R (di/dt) + i/C = 0. \tag{9.63}$$

Equation (9.63) is solved with the boundary conditions  $i(0+) = 0$  and  $di/dt(0+) = V_0/L$ , where  $t = 0$  is the switching time. The second condition follows from the fact that immediately after switching

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.24** Voltage on a load driven by a switched capacitor with series inductance as a function of  $\beta/\omega_0$ :  $L$  and  $C$  constant, load resistance ( $R$ ) varied;  $0 \leq \omega_0 t \leq 10$ . (a) Underdamped circuit ( $\beta/\omega_0 = 0.1$ ). (b) Critically damped circuit ( $\beta/\omega_0 = 1$ ). (c) Overdamped circuit ( $\beta/\omega_0 = 5$ ).

the total capacitor voltage appears across the inductor rather than the resistor (see Section 9.1).

The solution of Eq. (9.63) is usually written in three different forms, depending on the values of the following parameters:

$$\omega_0 = 1/\sqrt{LC}, \quad (9.64)$$

$$\beta = R/2L, \quad (9.65)$$

$$\omega_1 = \sqrt{\omega_0^2 - \beta^2},$$

$$\omega_2 = \sqrt{\beta^2 - \omega_0^2},$$

and

$$\delta = \tan^{-1}(\beta/\omega_1).$$

The solution with  $\beta < \omega_0$  is illustrated in Figure 9.24a. The circuit behavior is oscillatory. Energy is transferred back and forth between the inductor and capacitor at approximately the characteristic frequency,  $\omega_0$ . There is a small energy loss each oscillation, determined by the damping parameter

## Electrostatic Accelerators and Pulsed High Voltage

The circuit is *underdamped*. We shall study the underdamped LRC circuit in more detail when we consider rf accelerators. The time dependent current is

$$\begin{aligned} i(t) &= (CV_0/\cos\delta) \exp(-\beta t) [\omega_1 \sin(\omega_1 t - \delta) + \beta \cos(\omega_1 t - \delta)] \\ &\cong CV_0 \omega_0 \exp(-\beta t) \sin\omega_0 t \quad (\beta \ll \omega_0). \end{aligned} \quad (9.66)$$

The voltage on the load resistor is  $i(t)R$ .

The opposite extreme, *overdamping*, occurs when  $\beta > \omega_0$ . As indicated in Figure 9.24b, the circuit is dominated by the resistance and does not oscillate. The monopolar voltage pulse on the load rises in a time of approximately  $L/R$  and decays exponentially over a time  $RC$ . The current following switching in an overdamped circuit is

$$i(t) = [CV_0 (\beta^2 - \omega_0^2)/2\omega_0] [\exp(\omega_2 t) - \exp(-\omega_2 t)] \exp(-\beta t). \quad (9.67)$$

An LRC circuit is critically damped when  $\beta = \omega_0$  or

$$R_c = 2 \sqrt{L/C}. \quad (9.68)$$

The current for a critically damped circuit is

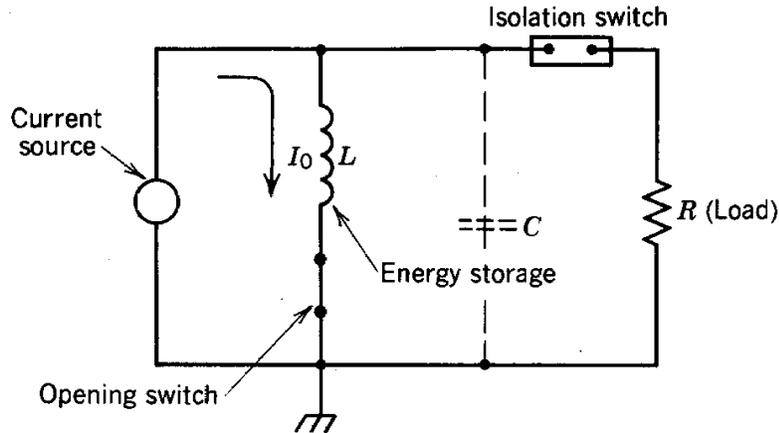
$$i(t) = \beta CV_0 (\beta t) \exp(-\beta t). \quad (9.69)$$

The time-dependent load voltage is plotted in Figure 9.24c.

The waveforms in Figures 9.24a, b, and c have the same values of  $L$  and  $C$  with different choices of  $R$ . Note that the transfer of energy from the capacitor to the load resistor is accomplished most rapidly for the critically damped circuit. Thus, the power extracted from a pulsed voltage modulator is maximum when  $R = R_c$ . The quantity  $2 \sqrt{L/C}$ , which has units of ohms, is called the characteristic impedance of the voltage modulator. Energy transfer is optimized when the load resistance is matched to the modulator impedance, as specified by Eq. (9.68). The peak power flow in a critically damped circuit occurs at a time  $t = 1/\beta$ . The maximum voltage at this time is  $V_{max} = 0.736V_0$ , the maximum current is  $0.368V_0/\sqrt{L/C}$ , and the maximum power in the load is  $0.271V_0^2/\sqrt{L/C}$ . The maximum power from this simple pulsed power generator is limited by the parasitic inductance of the circuit.

It is possible, in principle, to use an inductor for energy storage in a pulsed power circuit. A magnetic energy storage circuit is illustrated in Figure 9.25. In this case, a normally closed switch must be opened to transfer the energy to a load. We could also include the effects of parasitic capacitance in the circuit. Charging of this capacitance limits the current risetime in the load, analogous to the inductance in the capacitive storage circuit. The main advantage of magnetic

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.25** Pulse modulator with inductive energy storage and opening switch.

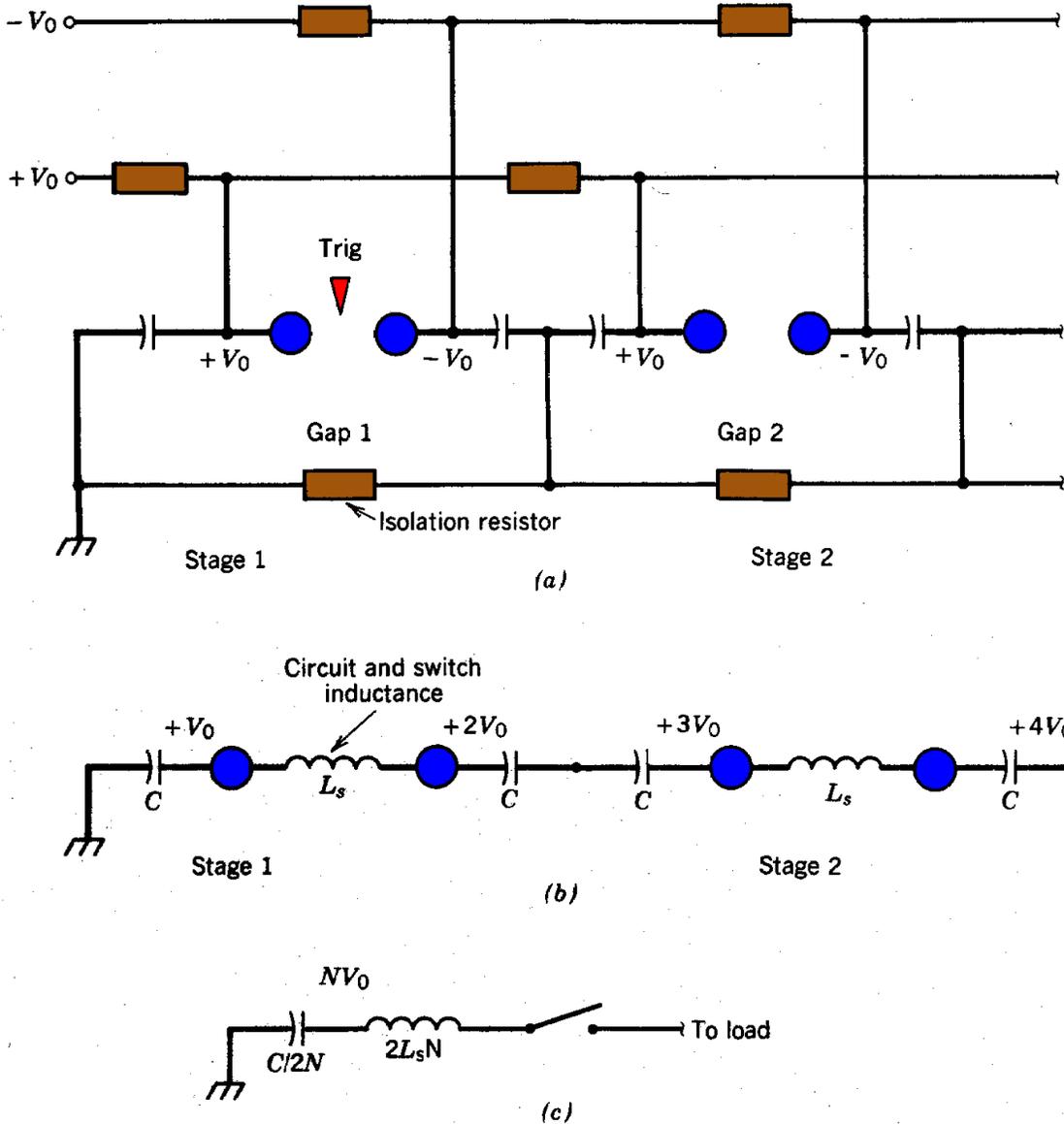
energy storage for, pulsed power applications is that a much higher energy density can be stored. Consider, for instance, a Mylar insulated capacitor with  $\epsilon/\epsilon_0 \cong 3$  and a field stress of 200 kV/cm. Applying Eq. (5.19), the electrical energy density is only  $10^4 \text{ J/m}^3$  ( $0.01 \text{ J/cm}^3$ ). In contrast, the energy density in a vacuum inductor with a field of 2 T (20 kG) is  $1.6 \times 10^6 \text{ J/m}^3$  ( $1.6 \text{ J/cm}^3$ ). The reason magnetic storage is not regularly used is the lack of a suitable fast opening switch. It is relatively easy to make a fast closing switch that conducts no current until a self-sustained discharge is initiated, but it is difficult to interrupt a large current and hold off the subsequent inductive voltage spike.

## 9.7 IMPULSE GENERATORS

The single-capacitor modulator is suitable for voltages less than 100 kV, but is seldom used at higher voltages. Transformer-based power supplies for direct charging are very large above 100 kV. Furthermore, it is difficult to obtain commercial high-energy-density capacitors at high voltage. Impulse generators are usually used for pulsed high voltage in the 0.1-10 MV range. These generators consist of a number of capacitors charged in parallel to moderate voltage levels ( $\sim 50 \text{ kV}$ ). The capacitors are switched to a series configuration by simultaneously triggered shorting switches. The voltages of the capacitors add in the series configuration.

We will consider two widely used circuits, the *Marx generator* [E. Marx, *Electrotech. Z.* **45**,652 (1925)] and the *LC generator*. Impulse generators that produce submicrosecond pulses require less insulation than a single capacitor with a steady-state charge at the full output voltage. The peak voltage is applied for only a short time, and breakdown paths do not have time to form in the liquid or gas insulation. High-field stress means that compact systems can be designed. Small systems have lower parasitic inductance and can therefore achieve higher output power.

Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.26** Marx generator. (a) Complete circuit diagram of a typical Marx generator showing charging lines. (b) Circuit model applicable for fast energy transfer following switching. (c) Simple lumped element circuit model for fast energy transfer neglecting electromagnetic wave phenomena.

A typical configuration for a Marx generator is illustrated in Figure 9.26. Positive and negative power supplies ( $\pm V_0$ ) are used, charging the capacitors as shown in Figure 9.26a. Charging current is carried to the capacitors through isolation resistors (or inductors) that act as open circuits during the fast output pulse. The capacitor stack is interrupted by high-voltage shorting switches. Gas-filled distortion spark gaps are ideal for this application. The trigger enters at the midplane of

## Electrostatic Accelerators and Pulsed High Voltage

the switch and is referenced to dc ground.

The circuit configuration immediately after switch shorting is shown in Figure 9.26b. The voltage across the load rapidly increases from 0 to  $2NV_0$ , where  $N$  is the number of switches and  $V_0$  is the magnitude of charge voltage on each capacitor. The series inductance shown arises mainly from the narrow discharge channels in the spark gaps. If we ignore voltage variations on time scales less than the dimension of the generator divided by the speed of light in the insulating medium, the inductances and capacitors can be lumped together as shown in Figure 9.26c. The Marx generator in the high-voltage phase is equivalent to the single capacitor modulator. Output to a resistive load is described by the equations of Section 9.6.

The total series inductance is proportional to the number of switches, while the series capacitance is  $C_0/2N$ . Therefore, the characteristic generator impedance is proportional to  $N$ . This implies that it is difficult to design high-voltage Marx generators with low characteristic impedance. High-energy density capacitors and short connections help lower the inductance, but the main limitation arises from the fact that the discharge current must flow through the inductive spark gap switches.

One favorable feature of the Marx generator is that it is unnecessary to trigger all the switches actively. If some of the spark gaps at the low-voltage end of the stack are shorted, there is overvoltage on the remaining gaps. Furthermore, the trigger electrodes of the spark gaps can be connected by circuit elements so that a trigger wave propagates rapidly through the generator. Using these techniques, pulsed voltages exceeding 10 MV have been generated in Marx generators with over 100 synchronous switches.

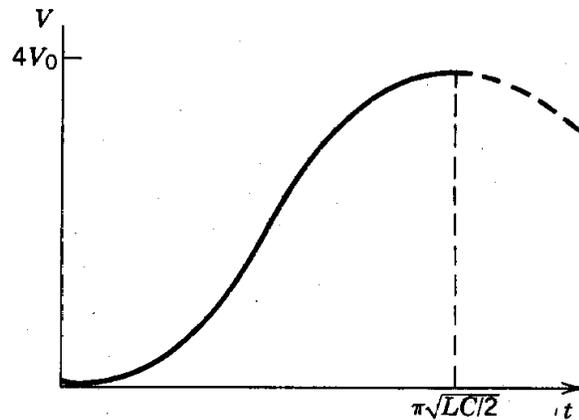
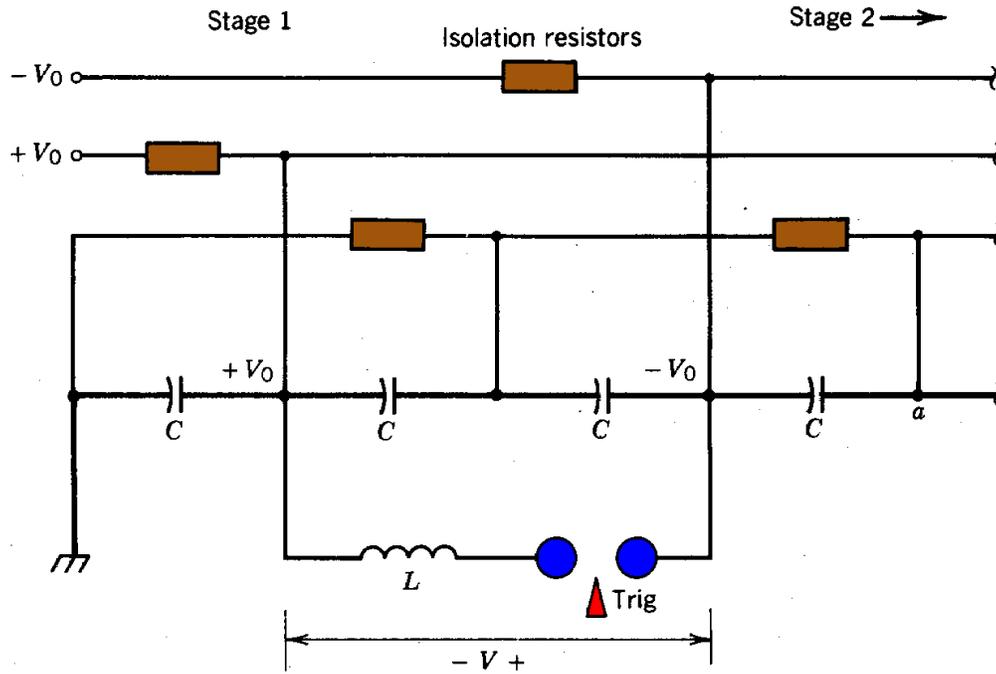
The  $LC$  generator, illustrated in Figure 9.27, is more difficult to trigger than the Marx generator, but it has lower characteristic impedance for the same output voltage. As in the Marx generator, a stack of capacitors is charged slowly in a parallel configuration by a positive-negative voltage supply (Fig. 9/27a). The main difference is that the switches are external to the main power flow circuit. Transition from a parallel to a series configuration is accomplished in the following way. Half of the capacitors are connected to external switched circuits with a series inductance. When the switches are triggered, each  $LC$  circuit begins a harmonic oscillation. After one half-cycle, the polarity on the switched capacitors is reversed. At this time, the voltages of all capacitors add in series, as shown in Figure 9.27b. The voltage at the output varies as

$$V_{out} = (2NV_0) [1 - \cos(\sqrt{2/lc} t)]. \quad (9.70)$$

where  $C$  is the capacitance of a single capacitor,  $N$  is the number of switches, and  $V_0$  is the magnitude of the charge voltage.

The load must be isolated from the generator by a high-voltage switch during the mode change from parallel to series. The isolation switch is usually a low-inductance spark gap. It can be actively triggered or it can be adjusted for self-breakdown near the peak output voltage. Energy transfer to the load should take place at  $t = \sqrt{LC}/2$  so that no energy remains in the external inductors. The transfer must be rapid compared to the mode change time or energy will return to

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.27** Circuit diagram of a typical  $LC$  generator, showing voltage at point  $a$  following switching through the external inductor.

the inductors, reducing the generator efficiency. Typically, an  $LC$  generator may have a mode change time of  $6 \mu\text{s}$  and a transfer time to the load of  $0.3 \mu\text{s}$ .

The equivalent circuit for the  $LC$  generator in the series state is the same as that for the Marx generator (Fig. 9.26c). The main difference is that the series inductances are reduced by elimination of the switches. The main disadvantages of the  $LC$  generator compared to the Marx generator are that the switching sequence is more complex, a low-inductance output switch is

required, and the circuit remains at high voltage for a longer time. Triggering one reversing circuit does not result in an overvoltage on the spark gaps of other sections. Therefore, all the switches in an LC generator must be actively fired with strong, synchronized trigger pulses.

## 9.8 TRANSMISSION LINE EQUATIONS IN THE TIME DOMAIN

Most accelerator applications for pulse modulators require a constant-voltage pulse. The critically damped waveform is the closest a modulator with a single capacitor and inductor can approach constant voltage. Better waveforms can be generated by modulators with multiple elements. Such circuits are called *pulse-forming networks* (PFNs). The transmission line is the continuous limit of a PFN. We shall approach the analysis of transmission lines in this section by a lumped element description rather than the direct solution of the Maxwell equations. Application of transmission lines as modulators is discussed in the following section. Discrete element PFNs are treated in Section 9.11.

We will derive the transmission line equations in the time domain. The goal is to find total voltage and current on the line as functions of time and position in response to specified inputs. The input functions have arbitrary time dependence, and may contain a number of frequency components. The frequency-domain analysis will be used in the study of rf accelerators which operate at a single frequency (Section 12.6). In the frequency-domain analysis, each harmonic component is treated separately. Voltage and current along the transmission line are described by algebraic equations instead of differential equations.

We will concentrate on the coaxial transmission line, illustrated in Figure 9.28. Properties of other common geometries are listed in Table 9.3. The coaxial line consists of an inner conducting

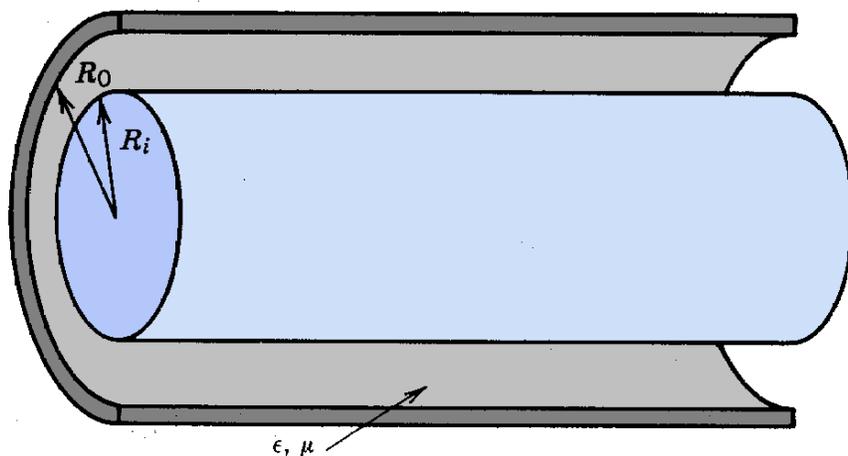
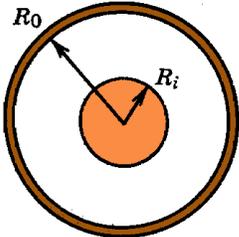
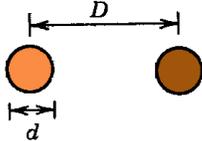
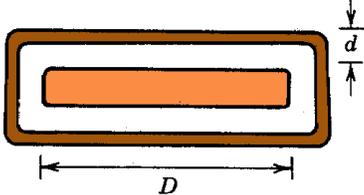


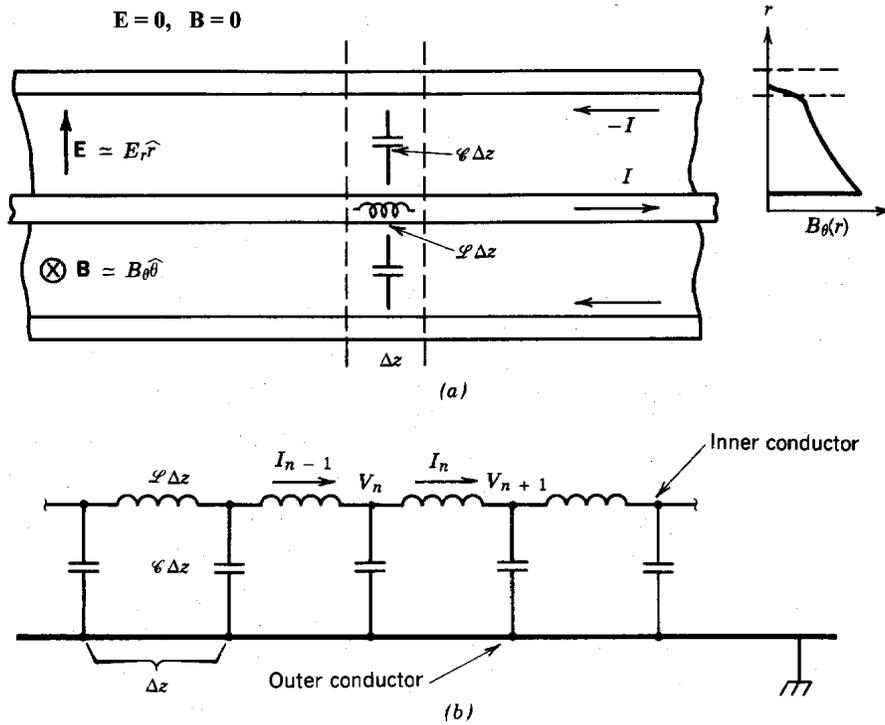
Figure 9.28 Section of a coaxial transmission line.

TABLE 9.3 Properties of Common Transmission Lines, TEM Modes <sup>a</sup>	
1 Coaxial transmission line:	 $\mathcal{C} = 2\pi\epsilon/\ln(R_o/R_i)$ $\mathcal{L} = (\mu/2\pi)\ln(R_o/R_i)$ $Z_0 = (\sqrt{\mu/\epsilon}/2\pi)\ln(R_o/R_i)$
2 Two-wire transmission line	 $\mathcal{C} = \pi\epsilon/\cosh^{-1}(D/d)$ $\mathcal{L} = \mu/\pi \cosh^{-1}(D/d)$ $Z_0 = (\sqrt{\mu/\epsilon}/\pi)\cosh^{-1}(D/d)$
3 Isolated parallel plates ( $d \ll D$ )	 $\mathcal{C} = \epsilon D/d$ $\mathcal{L} = \mu d/D$ $Z_0 = \sqrt{\mu/\epsilon}(d/D)$
4 Stripline ( $d \ll D$ )	 $\mathcal{C} = 2\epsilon D/d$ $\mathcal{L} = \mu d/2D$ $Z_0 = \sqrt{\mu/\epsilon}(d/2D)$

<sup>a</sup> $\mathcal{C}$  = capacitance per unit length (farads/meter);  $\mathcal{L}$  = inductance per unit length (henries/meter);  $Z_0$  = characteristic impedance (ohms).

cylinder (of radius  $R_i$ ) and a grounded outer cylinder ( $R_o$ ) separated by a medium with dielectric constant  $\epsilon$ . We assume a linear magnetic permeability,  $\mu$ . Figure 9.29a shows a sectional view of a line divided into differential elements of length ( $\Delta z$ ). Each element has a capacitance between the center and outer conductors proportional to the length of the element. If  $c$  is the capacitance per length, the capacitance of an element is  $c\Delta z$ . Magnetic fields are produced by current flow along the center conductor. Each differential element also has a series inductance,  $l\Delta x$  where  $l$  is the inductance per unit length. The circuit model of Figure 9.29b can be applied as a model of the transmission line. The quantities  $c$  and  $l$  for cylindrical geometry are given by

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**Figure 9.29** Coaxial transmission line. (a) Physical basis for lumped circuit element model of TEM wave propagation. (b) Lumped circuit element analog of a coaxial transmission line.

$$c = 2\pi\epsilon / \ln(R_o/R_i) \quad (F/m), \quad (9.71)$$

$$l = (\mu/2\pi) \ln(R_o/R_i) \quad (H/m). \quad (9.72)$$

Before solving the circuit of Figure 9.29b, we should consider carefully the physical basis for the correspondence between the circuit model and the coaxial line. The following observations about the nature of electric and magnetic fields in the line are illustrated in Figure 9.29a.

1. Fast-pulsed magnetic fields cannot penetrate into a good conducting material. We shall study this effect, the magnetic skin depth, in Section 10.2. Because there is no magnetic field inside the center conductor, the longitudinal current must flow on the outer surface.
2. The outer conductor is thick compared to the magnetic skin depth. Therefore, magnetic fields resulting from current flow on the inner conductor are not observed outside the line. Real current flowing on the outside of the inner conductor is balanced by negative current flow on the inside of the outer conductor. Magnetic fields are confined between the two cylinders.

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3. To an external observer, the outer conductor is an equipotential surface. There is no electrostatic voltage gradient along the conductor, and there is no inductively generated voltage since there are no magnetic fields outside the conductor. Furthermore, the inside surface of the outer conductor is at the same potential as the outside surface since there are no magnetic fields in the volume between the surfaces.
4. If the distance over which current on the inner conductor varies is large compared to  $(R_o - R_i)$ , then the only component of magnetic field is toroidal,  $B_\theta$ .
5. Similarly, if the voltage on the inner conductor varies over a long distance scale, then there is only a radial component of electric field.

Observations 1, 2, and 3 imply that we can treat the outer conductor as an ideal ground; voltage variations occur along the inner conductor. The two quantities of interest that determine the fields in the line are the current flow along the inner conductor and the voltage of the inner conductor with respect to the grounded outer conductor. Observations 4 and 5 define conditions under which electromagnetic effects can be described by the simple lumped capacitor and inductor model. The condition is that  $\omega \ll v/(R_o - R_i)$ , where  $\omega$  is the highest frequency component of the signal and  $v = 1/\sqrt{\epsilon\mu}$ . At higher frequency, complex electromagnetic modes can occur that are not well described by a single capacitor and inductor in a length elemental

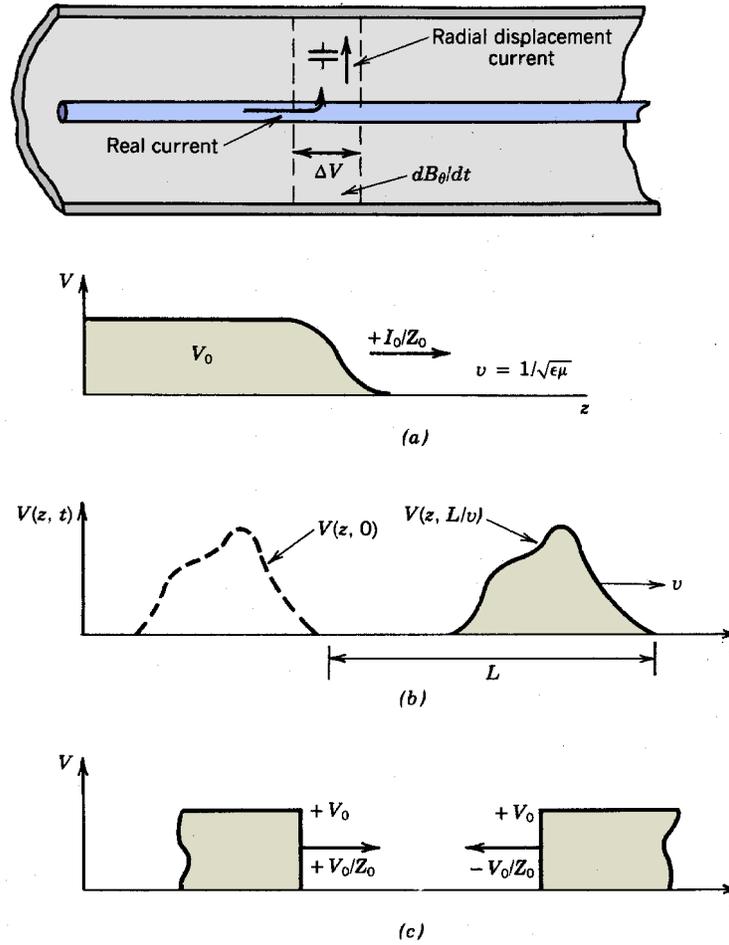
Voltage differences along the center conductor are inductive. They are supported by changes in magnetic flux in the region between the two cylinders. Differences in current along the center conductor result from displacement current between the inner and outer conductors. The interaction of voltage, real current, and displacement current is illustrated in Figure 9.30a. The figure indicates the motion of a step input in voltage and current down a transmission line. The magnetic field behind the front is constant; the flux change to support the voltage differences between the inner and outer conductors comes about from the motion of the front. There is a rapid change of voltage at the pulse front. This supports a radial displacement current equal to the current of the pulse. Current returns along the other conductor to complete the circuit. The balance between these effects determines the relationship between voltage and current and the propagation velocity of the pulse.

Referring to Figure 9.29b, the voltage at point  $n$  is equal to the total current that has flowed into the point divided by the capacitance. Using the sign convention shown, this statement is expressed by

$$\int dt (I_{n-1} - I_n) / (c\Delta z) = V_n. \quad (9.73)$$

Taking the derivative,

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.30** Coaxial transmission line. (a) Distribution of voltage, real current, and displacement current for a sharp rising step pulse propagating along a transmission line. (b) Distributions of voltage on a transmission line at time  $t = 0$  (dashed line) and time  $t = L/v$  (solid line). (c) Polarity conventions for positive voltage impulses traveling in the  $+z$  and  $-z$  directions.

$$-c\Delta z \left( \frac{\partial V_n}{\partial t} \right) = I_n - I_{n-1}. \quad (9.74)$$

The relationship is a partial differential equation because we are viewing the change in voltage with time at a constant position. The difference in voltage between two points is the inductive voltage between them, or

$$l\Delta z \left( \frac{\partial I_n}{\partial t} \right) = V_n - V_{n+1}. \quad (9.75)$$

If  $\Delta z$  becomes small, the discrete voltages approach a continuous function,  $V(z, t)$ . The voltage difference between two points can be approximated in terms of this function as

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$$V_{n+1} \cong V_n + [\partial V(z_n, t)/\partial z] \Delta z. \quad (9.76)$$

Similarly,

$$I_{n-1} \cong I_n - [\partial I(z_n, t)/\partial z] \Delta z. \quad (9.77)$$

Substituting the results of Eqs. (9.76) and (9.77) into Eqs. (9.74) and (9.75) gives the continuous partial differential equations

$$\partial V/\partial z = -l (\partial I/\partial t), \quad (9.78)$$

$$\partial I/\partial z = -c (\partial V/\partial t). \quad (9.79)$$

Equations (9.78) and (9.79) are called the *telegraphist's equations*. They can be combined to give wave equations for  $V$  and  $I$  of the form

$$\partial^2 V/\partial z^2 = (lc) (\partial^2 V/\partial t^2). \quad (9.80)$$

Equation (9.80) is a mathematical expression of the properties of a transmission line. It has the following implications:

1. It can easily be verified that any function of the form

$$V(z, t) = F(t \pm z/v) \quad (9.81)$$

is a solution of Eq. (9.80) if

$$v = 1/\sqrt{lc}. \quad (9.82)$$

The spatial variation of voltage along the line can be measured at a particular time  $t$  by an array of probes. A measurement at a time  $t + \Delta t$  would show the same voltage variation but translated a distance  $v\Delta t$  either upstream or downstream. This property is illustrated in Figure 9.30b. Another way to phrase this result is that a voltage pulse propagates in the transmission line at velocity  $v$  without a change in shape. Pulses can travel in either the  $+z$  or  $-z$  directions, depending on the input conditions.

2. The current in the center conductor is also described by Eq. (9.80) so that

$$I(z, t) = G(t \pm z/v). \quad (9.83)$$

Measurements of current distribution show the same velocity of propagation.

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3. The velocity of propagation in the coaxial transmission line can be found by substituting Eqs. (9.71) and (9.72) into Eq. (9.82):

$$v = 1/\sqrt{\epsilon\mu} = c/\sqrt{(\epsilon/\epsilon_o)(\mu/\mu_o)}. \quad (9.84)$$

This velocity is the speed of light in the medium. The geometric factors in  $c$  and  $l$  cancel for all transmission lines so that the propagation velocity is determined only by the properties of the medium filling the line.

4. Inspection of Eqs. (9.78) and (9.79) shows that voltage is linearly proportional to the current at all points in the line, independent of the functional form of the pulse shape. In other words,

$$V = I Z_o, \quad (9.85)$$

where  $Z_o$  is a real number. The quantity  $Z_o$  is called the *characteristic impedance* of the line. Its value depends on the geometry of the line. The characteristic impedance for a coaxial transmission line is

$$Z_o = \sqrt{l/c} = \sqrt{\mu/\epsilon} \ln(R_o/R_i)/2\pi. \quad (9.86)$$

Rewriting Eq. (9.86) in practical units, we find that

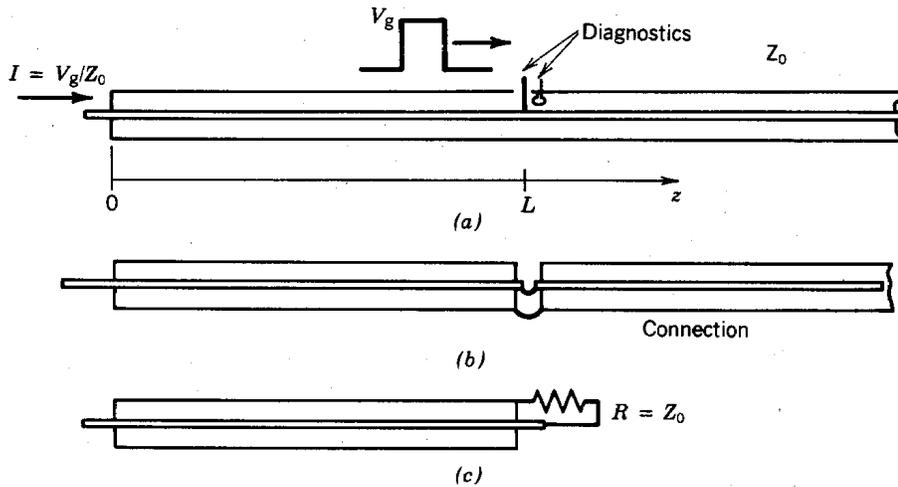
$$Z_o = \frac{60 \ln(R_o/R_i) \sqrt{\mu/\mu_o}}{\sqrt{\epsilon/\epsilon_o}} \quad (\Omega). \quad (9.87)$$

5. Polarities of voltage and current often cause confusion. Conventions are illustrated in Figure 9.30c. A positive current is directed in the  $+z$  direction; a negative current moves in the  $-z$  direction. A positive-going current waveform creates a magnetic flux change that results in a positive voltage on the center conductor.

## 9.9 TRANSMISSION LINES AS PULSED POWER

A transmission line is a distributed capacitor. Energy is stored in the line when the center conductor is charged to high voltage. Transmission lines have the property that they can produce a constant-voltage output pulse when discharged into a resistive load. In order to understand this, we shall first consider the properties of signal propagation on finite length lines with resistive loads at an end. A load at the end of a transmission line is called a *termination*.

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**Figure 9.31** Matched terminations of a transmission line. (a) Infinite length line with characteristic impedance  $Z_0$ . (b) Finite length line connected to a semiinfinite length line with the same characteristic impedance. (c) Transmission line terminated by a matched resistor ( $R = Z_0$ ).

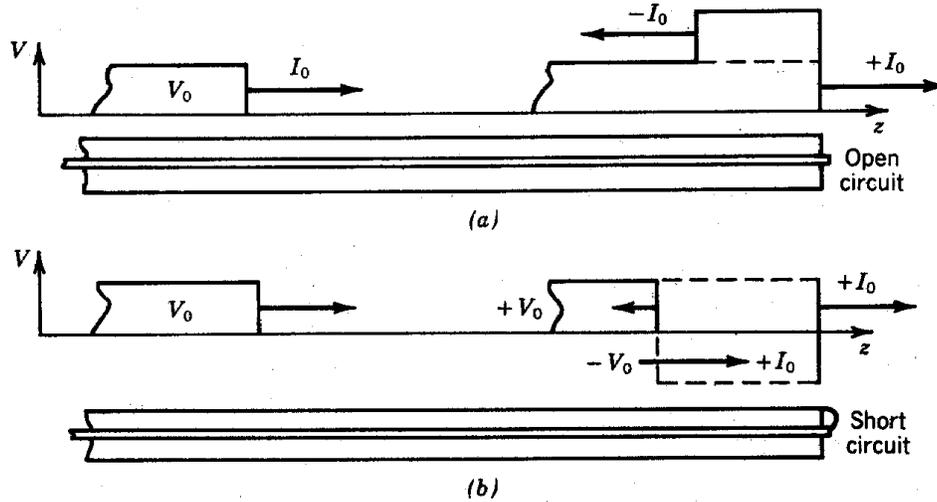
Figure 9.31 shows a transmission line extending from  $z = 0$  to infinity driven by a voltage generator. The generator determines the entrance boundary condition on the line:  $V_g(z = 0, t)$ . The generator supplies a current  $V_g/Z_0$ . The assumption of an infinite line means, that there are no negatively-directed waves. There is a positively-directed wave produced by the generator. A voltage probe a distance  $L$  from the generator measures a signal  $V_s(t') = V_g(t' - L/v)$ . Similarly a current probe gives a signal  $I_s(t') = V_g(t' - L/v)/Z_0$ .

Assume that the line is split at the point  $L$  and the pieces are connected together with a good coaxial connector. This change makes no difference in wave propagation on the line or the signals observed at  $L$ . Proceeding another step, the infinite length of line beyond the split could be replaced with a resistor with value  $R = Z_0$ . The important point is that, in terms of observations at  $L$ , wave propagation with the resistive termination is indistinguishable from that with the infinite line. The boundary condition at the connection point is the same,  $V(L, t) = I(L, t)Z_0$ . In both cases, the energy of the pulse passes through the connector and does not return. In the case of the line, pulses propagate to infinity. With the resistor, the pulse energy is dissipated. The resistor with  $R = Z_0$  is a *matched termination*.

We must consider the properties of two other terminations in order to understand the transmission line as an energy storage element. One of them is an open circuit, illustrated in Figure 9.12a. Assume the voltage generator produces a sharp rising step pulse with voltage  $+V_0$  and current  $+I_0$ . When the step function reaches the open circuit, it can propagate no further. The open-circuit condition requires that a probe at position  $z = L$  measures zero current. This occurs if the boundary reflects the original wave as shown, giving a pulse propagating in the negative  $z$  direction with positive voltage and negative current. When this pulse is added to the positive-going pulse from the generator, the net current is zero and the net voltage is  $+2V_0$ .

Similar considerations apply to the short-circuit termination (Fig. 9.32b). A voltage probe at  $z = L$  always measures zero voltage. The interaction of the incoming wave with a short-circuit

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**Figure 9.32** Reflection of step-function pulses at a transmission line termination. (a) Open circuit. (b) Short circuit.

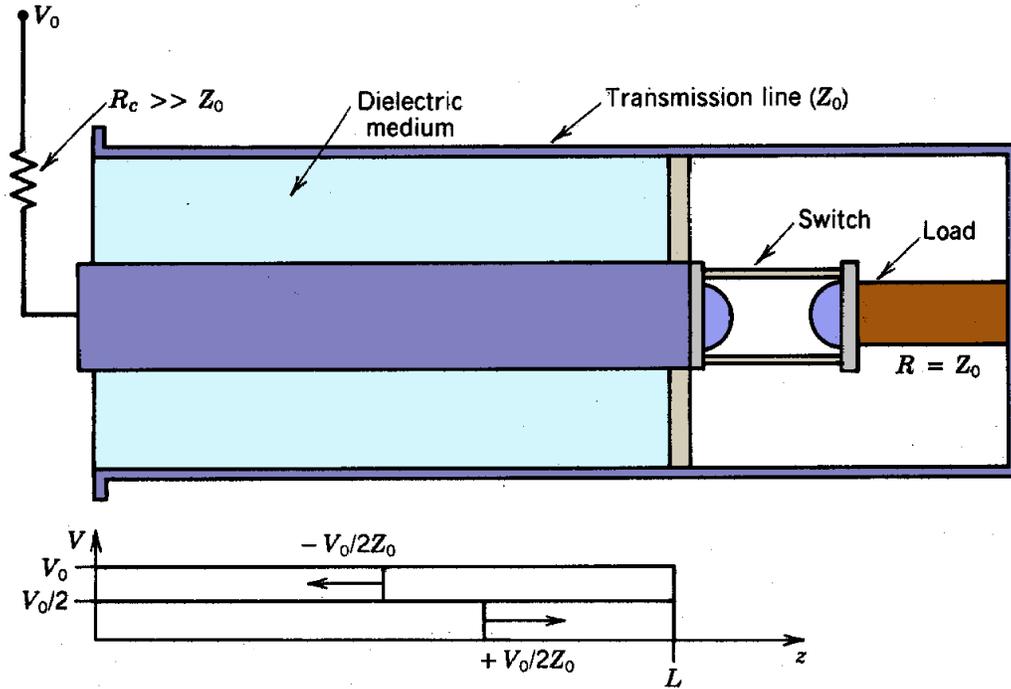
termination generates a negative-going wave with negative voltage and positive current. In other words, the termination acts as an inverting transformer. These reflection properties can be confirmed by a direct numerical solution of Eqs. (9.78) and (9.79) with appropriate boundary conditions.

The circuit for a transmission line pulsed voltage modulator is shown in Figure 9.33. The center conductor of a line of length  $L$  is charged to high voltage by a power supply. The supply is connected through a large resistance or inductance; the connection can be considered an open circuit during the output pulse. The other end of the line is connected through a shorting switch to the load. We assume the load is a matched termination,  $R = Z_0$ .

Consider, first, the state of the charged line before switching. The center conductor has voltage  $V_0$ . There is no net current flow. Reference to Figure 9.33 shows that the standing voltage waveform of this state can be decomposed into two oppositely directed propagating pulses. The pulses are square voltage pulses with length  $L$  and magnitude  $\frac{1}{2}V_0$ . The positively-directed pulse reflects from the switch open-circuit termination to generate a negatively-directed pulse that fills in behind the original negative-going pulse. The same process occurs at the other open-circuit termination; therefore, the properties of the line are static.

The resolution of a static charge into two traveling pulses helps in determining the time-dependent behavior of the circuit following switching. We are free to choose the boundaries of the positive and negative pulses at any location; let us assume that they are at  $z = 0$  and  $z = L$  at the time of switching,  $t = 0$ . Following switching, the positive-going pulse travels into the termination while the negative pulse moves away from it. The positive pulse produces a flat-top voltage waveform of duration  $\Delta t = L/v$  and magnitude  $\frac{1}{2}V_0$  in the load. At the same time, the negative-going pulse reflects from the open circuit at the charging end and becomes a

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**Figure 9.33** Coaxial transmission line used as a square-pulse generator.

positive-going pulse following immediately behind the original. The negative pulse deposits its energy in the load during the time  $L/v \leq t \leq 2L/v$ . In summary, the discharge of a transmission line through a shorting switch into a matched resistive load produces a constant-voltage pulse. The magnitude of the pulse is  $\frac{1}{2}V_0$  and the duration is

$$\Delta t_p = 2L/v. \tag{9.88}$$

where  $v$  is the pulse propagation velocity.

The total capacitance of a transmission line of length  $L$  can be expressed in terms of the single transit time for electromagnetic pulses ( $\Delta t = L/v$ ) and the characteristic impedance as

$$C = \Delta t/Z_0. \tag{9.89}$$

It is easily demonstrated that energy is conserved when the line is discharged into a matched termination by comparing  $CV_0^2$  to the time integral of power into the load. The total series inductance of a transmission line is

$$L = Z_0 \Delta t. \tag{9.90}$$

Transformer oil is a common insulator for high-voltage transmission lines. It has a relative dielectric constant of 3.4. The velocity of electromagnetic pulses in oil is about 0.16 m/ns. Voltage

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hold-off in a coaxial transmission line is maximized when  $R_i/R_o = 1/e$ . For oil this translates into a  $33 \Omega$  characteristic impedance [Eq. (9.87)]. Purified water is used as a transmission line energy storage medium in high-power density pulsed modulators because of its high relative dielectric constant ( $\epsilon/\epsilon_o \cong 81$ ). Water is conductive, so that water lines must be pulse charged. In comparison with oil, a water line with a  $1/e$  radius ratio can drive a  $6.8 \Omega$  load. For the same charge voltage, the energy density in a water line is 24 times higher than in an oil line.

### 9.10 SERIES TRANSMISSION LINE CIRCUITS

Two features of the transmission line pulse modulator are often inconvenient for high-voltage work. First, the matched pulse has an amplitude only half that of the charge voltage. Second, the power transfer switch must be located between the high-voltage center conductor and the load. The switch is boosted to high voltage; this makes trigger isolation difficult. The problems are solved by the Blumlein transmission line configuration [A. D. Blumlein, U.S. Patent No. 2,465,840 (1948)]. The circuit consists of two (or more) coupled transmission lines. Fast-shortening switches cause voltage reversal in half the lines for a time equal to the double transit time of electromagnetic pulses. The result is that output pulses are produced at or above the dc charge voltage, depending on the number of stages. The Blumlein line circuit is the distributed element equivalent of the LC generator.

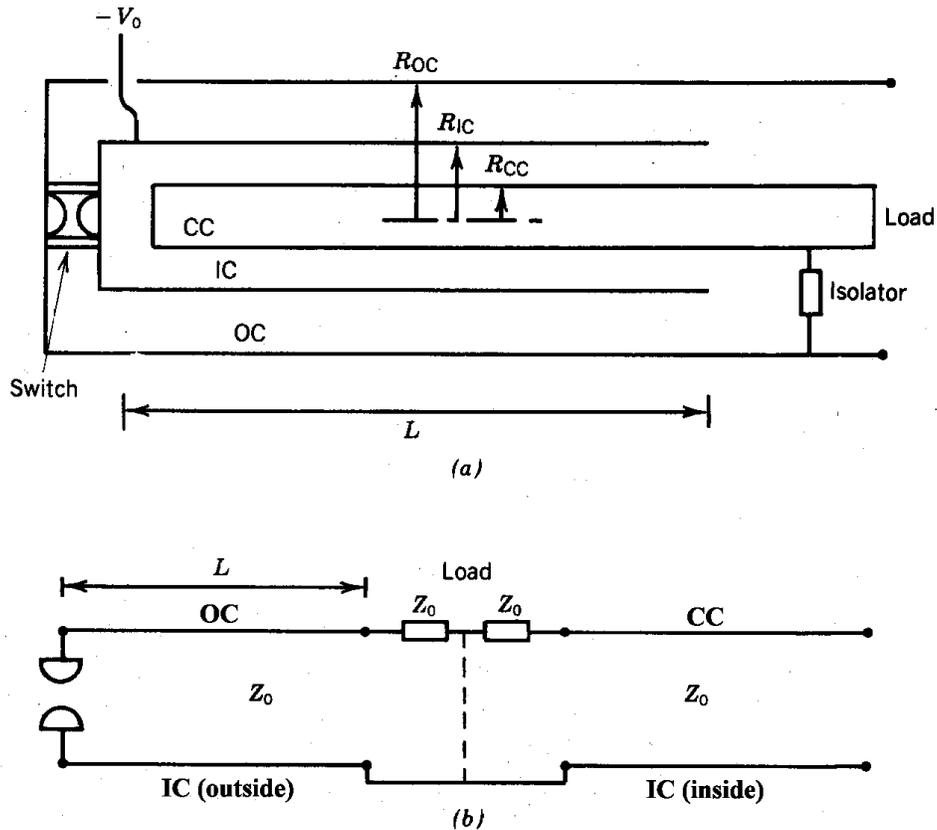
We shall analyze the two-stage transmission line driving a matched resistive load. A circuit with nested coaxial transmission lines is illustrated in Figure 9.34a. The three cylinders are labeled *OC* (outer conductor), *IC* (intermediate conductor), and *CC* (center conductor). The diameters of the cylinders are usually chosen so that the characteristic impedance of the inner line is equal to that of the outer line. This holds when  $R_{OC}/R_{IC} = R_{IC}/R_{CC}$ . We neglect end effects and assume that both lines have the same electromagnetic transit time.

A high-voltage feed penetrates the outer cylinder to charge the intermediate cylinder. The center conductor is connected to ground through an isolation element. The isolator acts as a short circuit over long times but approximates an open circuit during the output pulse. It is usually a simple inductor, although we shall investigate a more complex isolator when we study linear induction accelerators (Section 10.5). A shorting switch between the *IC* and *OC* is located at the end opposite the load.

The equivalent circuit of the two-stage Blumlein line is shown in Figure 9.34b. In order to analyze the pulse output of the circuit, we make the following assumptions:

1. The middle conductor is taken as a reference to analyze traveling voltage pulses.
2. The isolation element is an ideal open circuit during the pulse output.

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.34** Nested coaxial transmission lines in the Blumlein configuration. (a) Schematic view and definition of quantities. (b) Equivalent circuit.

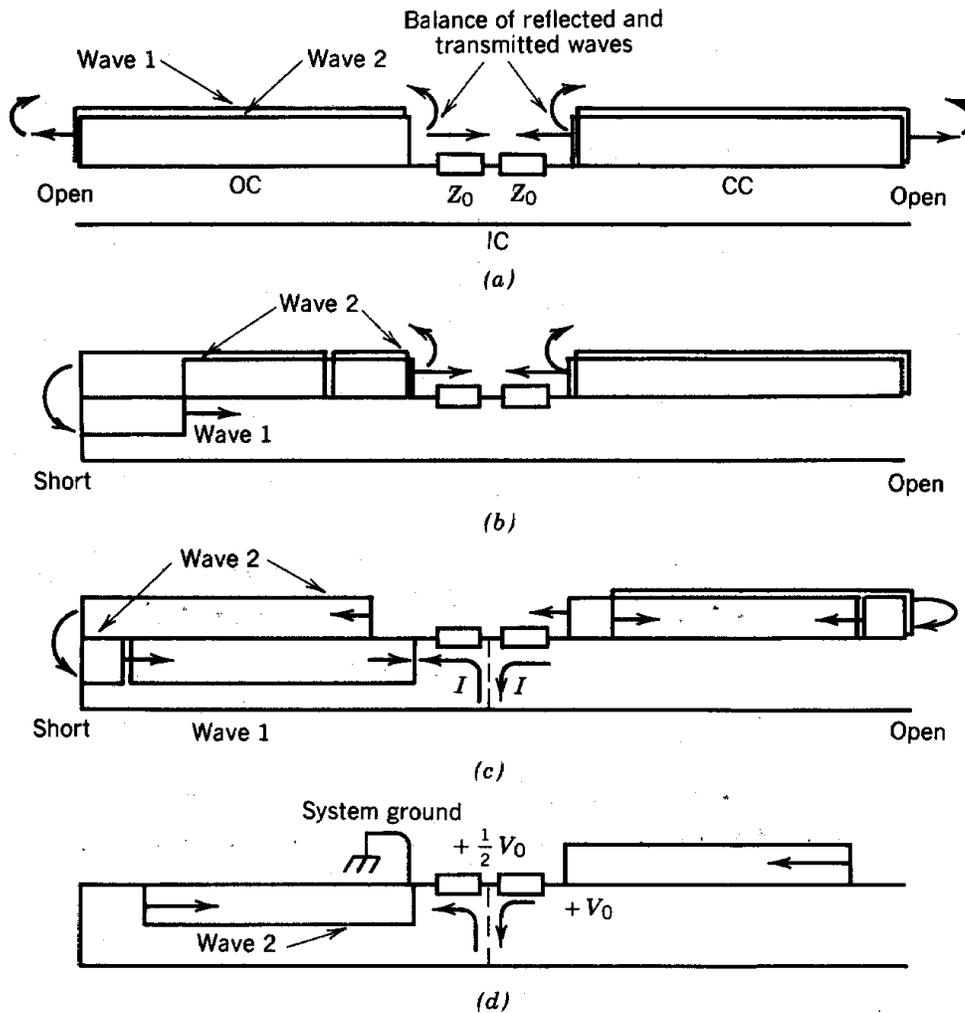
3. The load resistance is  $Z_1 = 2Z_0$ , where  $Z_0$  is the characteristic impedance of the individual nested lines.

4. An imaginary connection is attached from the intermediate conductor to the midpoint of the load during the output pulse. This connection is indicated by dotted lines in Figure 9.34b.

In the steady state, the intermediate conductor is charged to  $-V_0$ . The other two electrodes have positive voltage relative to the intermediate conductor. The static charge can be represented in the inner and outer lines as two oppositely-directed positive voltage pulses of length  $L$  and magnitude  $\frac{1}{2}V_0$ . Pulses arriving at the end connected to the load are partially reflected and partially transmitted to the other line. Transmission is the same in both directions so that a steady state is maintained. Assume that the boundaries of positive and negative pulses are aligned as shown in Figure 9.35a at switching time  $t = 0$ .

The following events take place when the shorting switch is activated:

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.35** Propagation and reflection of step-function pulses following switching of a Blumlein line. (a) Resolution of static charge voltage prior to switching into interleaved traveling pulses. (b) Pulse polarities and directions during interval  $0 < t < L/v$ . (c) Pulses during interval  $L/v < t < 2L/v$ . (d) Pulses during interval  $2L/v < t < 3L/v$ .

1. There is a time lag before information about the shorting is communicated the load. During this time, the  $+z$ -directed pulse in the outer line and the  $-z$ -directed pulse in the inner line continue to move toward the load. The balance of transmission and reflection causes a positive-polarity reflected pulse to move backward in each line as though the midpoint were an open circuit (Fig. 9.35b).

2. The  $+z$ -directed pulse in the inner line moves to the open-circuit end opposite the load and is reflected with positive polarity. The major activity in the circuit is associated with the  $-z$ -directed pulse on the outer line. This pulse encounters the short circuit and reflects with negative polarity.

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3. The negative-polarity pulse arrives at the load at  $\Delta t = L/v$ . We make the imaginary connection at this time. Power from the positive pulse in the inner line and the negative pulse in the outer line is deposited in the two matched loads (Fig. 9.35c). The voltages on the loads have magnitude  $\frac{1}{2}V_o$  and the same polarity; therefore, there is a net voltage of  $V_o$  between the connection points of the center and outer conductors to the load.
4. The situation of Figure 9.35c holds for  $\Delta t < t < 2\Delta t$ . During this time, the other pulse components in the two lines reflect from the boundaries opposite the load. The polarity of the reflected wave is positive in the inner line and negative in the outer line.
5. The pulses that originally moved away from the load deposit their energy during the time  $2\Delta t < t < 3\Delta t$  (Fig. 9.35d).
6. Inspection of Figures 9.35c and d shows that currents from the two lines are equal and opposite in the imaginary conductor during the output pulse. Because the connection carries no current, we can remove it without changing the circuit behavior.

The above model resolves the Blumlein line into two independent transmission lines driving a series load. The voltage in one line is reversed by reflection at a short circuit. Voltages that cancel in the static state add in the switched mode. In summary, the Blumlein line circuit has the following characteristics:

1. An output voltage pulse of  $V_o$  is applied to a matched load between the center conductor and the outer conductor for the double transit time of an individual line,  $2L/v$ .
2. The voltage pulse is delayed from the switch time by an interval  $L/v$ .
3. The matched impedance for a two-stage Blumlein line is  $2Z_o$ .
4. A negatively charged intermediate conductor results in a positive output pulse when the switch is located between the intermediate and outer conductors. The output pulse is negative if a shorting switch is located between the intermediate and center conductors.

The Blumlein line configuration is more difficult to construct than a simple transmission line. Furthermore, the Blumlein line has no advantage with respect to energy storage density. Neglecting the thickness of the middle conductor and its voltage grading structures, it is easy to show that the output impedance, stored energy density, pulse length, and output voltage are the same as that for an equal volume transmission line (with  $R_i = R_{CC}$  and  $R_o = R_{OC}$ ) charged to  $2V_o$ . The main advantage of the Blumlein line is that requirements on the charging circuit are relaxed. It is much less costly to build a 1-MV Marx generator than 2-MV generator with the same stored energy. Furthermore, in the geometry of Figure 9.34, the switch can be a trigatron with the triggered electrode on the ground side. This removes the problem of trigger line isolation.

### 9.11. PULSE-FORMING NETWORKS

Transmission lines are well suited for output pulselengths in the range  $5 \text{ ns} < \Delta t_p < 200 \text{ ns}$ , but they are impractical for pulselengths above  $1 \mu\text{s}$ . Discrete element circuits are usually used for long pulselengths. They achieve better output waveforms than the critically damped circuit by using more capacitors and inductors. Discrete element circuits that provide a shaped waveform are called *pulse-forming networks*.

The derivations of Section 9.8 suggest that the circuit of Figure 9.36 can provide a pulse with an approximately constant voltage. A transmission line is simulated by a finite number  $N$  of inductor-capacitor units. Following the derivation of Section 9.8, the resistance of a matched load is

$$Z_o = \sqrt{L/C}. \tag{9.91}$$

The quantities  $L$  and  $C$  are the inductance and capacitance of discrete elements. We shall call  $Z_o$  the impedance of the PFN. The single transit time of an electromagnetic pulse through the network is approximately  $N\sqrt{LC}$ . The output voltage pulse has average magnitude  $\frac{1}{2}V_o$  and duration  $\Delta t_p \cong 2N\sqrt{LC}$ .

The output pulse of a five-element network into a matched resistive load is shown in Figure 9.37. Although the general features are as expected, there is substantial overshoot at the beginning of the pulse and an undershoot at the end. In addition, there are voltage oscillations during the pulse. In some applications, these imperfections are not tolerable. For instance, the pulse modulator may be used to drive an ion injector where the beam optics depends critically on the voltage.

A Fourier analysis of the circuit of Figure 9.36 indicates the basis for the poor pulse shaping. The circuit generates  $N$  Fourier components with relative amplitudes optimized to replicate a sharp-edged square voltage pulse. In the Fourier series expansion of a square pulse, the magnitudes of the terms of order  $n$  decrease only as the inverse of the first power of  $n$ ,  $a_n \sim 1/n$ . Thus, many terms are needed for an accurate representation. In other words, a large number of

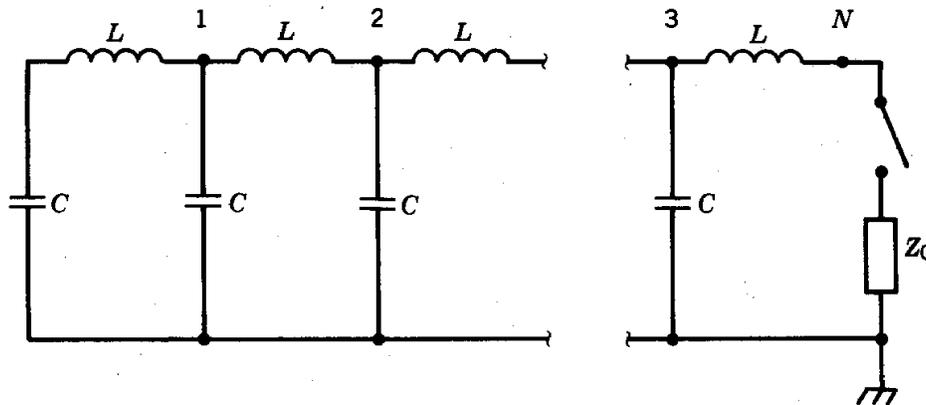
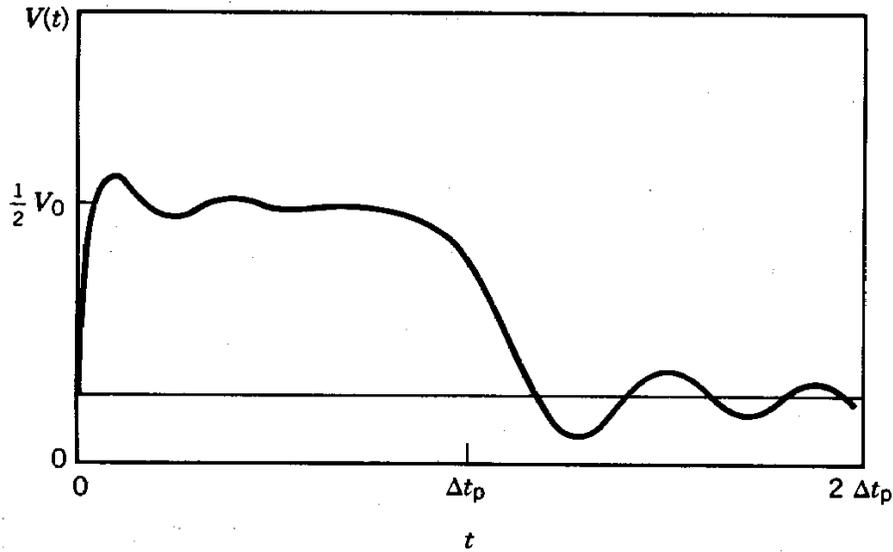


Figure 9.36 Pulse-forming network; lumped element approximation of a transmission line.



**Figure 9.37** Load voltage, five-element PFN discharged into a matched load.

elements is needed in the circuit of Figure 9.36 for a relatively constant output voltage. Such a division increases the size and cost of the modulator.

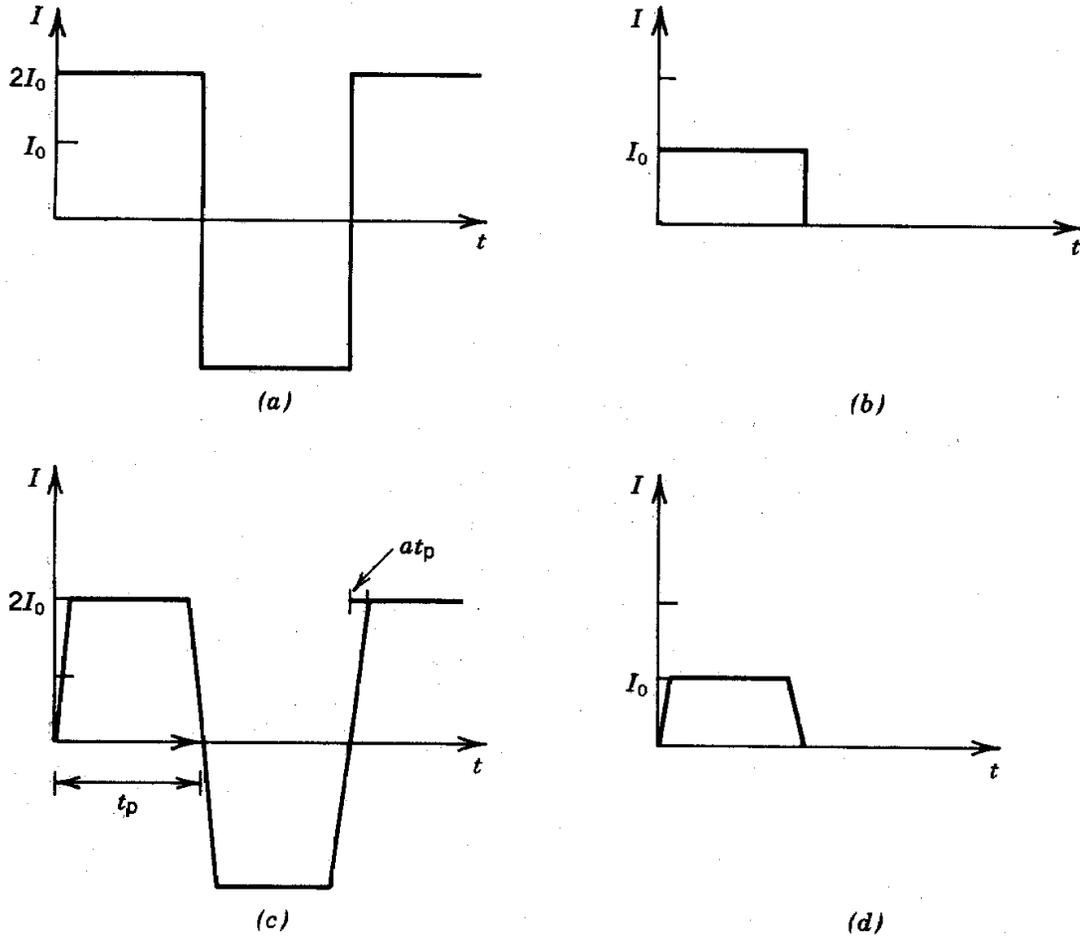
A different approach to the PFN, developed by Guillemin [E.A. Guillemin, **Communications Networks, Vol. II**, (Wiley, New York, 1935)] provides much better pulse shaping with fewer elements. He recognized that the slow convergence of the network of Figure 9.36 is a consequence of approximating a discontinuous waveform. In most applications, the main concern is a good voltage flat-top, and gradual voltage variation on the rise and fall of the pulse can be tolerated. The key is to work in reverse from a smooth waveform to derive a generating circuit.

We can utilize a Fourier series analysis if we apply the following procedure. Consider first an ideal transmission line discharged into a short-circuit load (Fig. 9.38a). The current oscillates between  $+2I_o$  and  $-2I_o$ , where  $I_o$  is the output current when the line is discharged into a matched load,  $I_o = V_o/2Z_o$ . The periodic bipolar waveform can be analyzed by a Fourier series. When the circuit is connected to a matched load, there is a single square pulse (Fig. 9.38b). By analogy, if we could determine a circuit that produced a different bipolar current waveform with peak amplitude  $2I_o$ , such as the trapezoidal pulse of Figure 9.38c, then we expect that it would produce a trapezoidal pulse (Fig. 9.38d) of amplitude  $I_o$  when connected to a resistance  $R = V_o/2I_o$ . We shall verify this analogy by direct computation.

It remains to determine a circuit that will produce the waveform of Figure 9.38c when discharged into a short. The Fourier series representation of a trapezoidal current pulse with magnitude  $2I_o$ , total length  $\Delta t_p$  and rise and fall times  $a\Delta t_p$  is

$$i(t) = 2I_o \sum_n \left( \frac{4}{n\pi} \right) \left( \frac{\sin(n\pi a)}{n\pi a} \right) \sin \left( \frac{n\pi t}{\Delta t_p} \right). \quad (9.92)$$

## Electrostatic Accelerators and Pulsed High Voltage



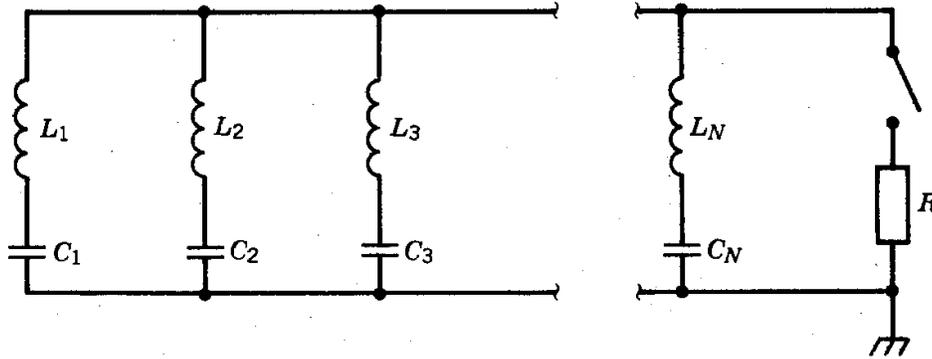
**Figure 9.38** Design concept for Guillemin networks. (a) Output current of an ideal transmission line discharged into a short circuit. (b) Output current of an ideal transmission line discharged into a matched resistor. (c) Trapezoidal pulse of a Guillemin network discharged into a short circuit. (d) Guillemin network discharged into a matched resistor.

Consider the circuit of Figure 9.39. It consists of a number of parallel  $LC$  sections. If the PFN is discharged into a short circuit, the current flow through each of the sections is independent of the others. This occurs because the voltage across each section is zero. The current in a particular section after switching is

$$i_n(t) = i_{n0} \sin \left( \frac{t}{\sqrt{L_n C_n}} \right) \quad (9.93)$$

We choose the inductance and capacitance of sections so that their free harmonic oscillations are at the frequency of the Fourier components in Eq. (9.92). In other words,

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.39** Basic Guillemin network.

$$\sqrt{L_1 C_1} = \Delta t_p / \pi, \quad \sqrt{L_2 C_2} = \Delta t_p / 2\pi, \quad \sqrt{L_3 C_3} = \Delta t_p / 3\pi, \quad \dots \quad (9.94)$$

We know from our study of the undamped  $LC$  circuit (Section 9.6) that the magnitude of the current flowing through any one of the sections after switching is

$$i_{n0} = V_0 / \sqrt{L_n / C_n}. \quad (9.95)$$

If  $Z_o$  is the desired characteristic impedance, the magnitudes are matched to the Fourier series if

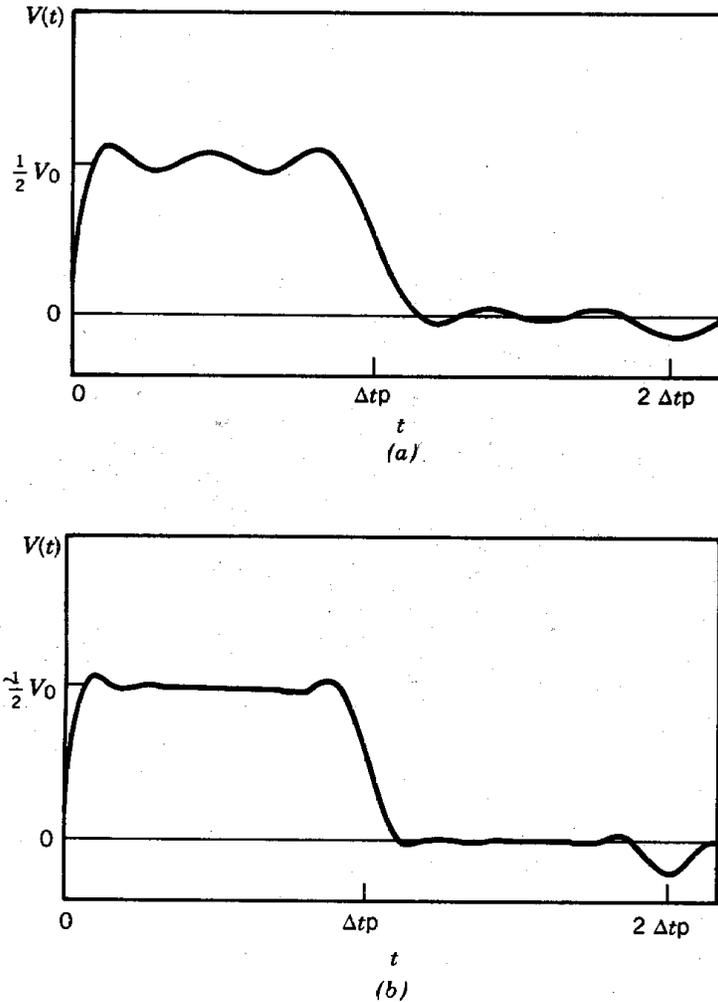
$$V_0 / \sqrt{L_n / C_n} = (V_0 / Z_o) (4/n\pi) [\sin(n\pi a) / n\pi a], \quad (9.96)$$

where we have substituted  $V_0 / Z_o = 2I_o$ . Equations (9.94) and (9.96) can be solved to give the appropriate component values of the PFN:

$$L_n = (Z_o \Delta t_p / 4) / [\sin(n\pi a) / n\pi a], \quad C_n = (4 \Delta t_p / n^2 \pi^2 Z_o) / [\sin(n\pi a) / n\pi a]. \quad (9.97)$$

Figure 9.40 shows a voltage pulse on a matched resistor ( $R = Z_o$ ) for three and five-element Guillemin networks using the circuit values determined from Eqs. (9.97). Note the improvement compared to the pulse shape of Figure 9.37. Equation (9.92) shows that the Fourier expansion converges as  $1/N^2$ . A better flat-top can be obtained by approximating smoother pulses. For instance, the series converges as  $1/N^3$  if the pulse is assumed to have parabolic edges with no discontinuity in slope. Alternate circuits to that of Figure 9.39 can be derived. The main disadvantage of the Guillemin network is that the capacitors all have different values. It is usually difficult to procure commercial high-voltage capacitors with the required capacitances.

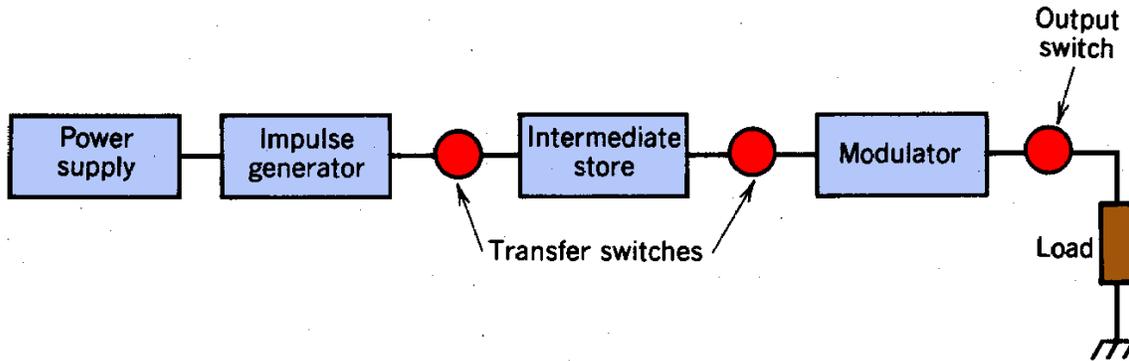
## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.40** Load voltage, Guillemin network discharged into a matched resistor. (a) Three-element network. (b) Five-element network.

## 9.12 PULSED POWER COMPRESSION

Electron and ion beams in the megampere range have been generated in pulsed power diodes [see T. H. Martin and M. F. Rose, Eds., **Proc. 4th IEEE Pulsed Power Conference**, (IEEE 83CH1908-3, Piscataway, New Jersey, 1983)]. Such beams have, application to inertial fusion and studies of materials at high temperature, pressure, and radiation levels. High-power pulse modulators are needed to drive the diodes. Pulsed power research is largely centered on extending the limits of the output power of voltage generators. At present, single-unit generators have been built that can apply power in the TW ( $10^{12}$  W) range to a load. Typical output parameters of such a modulator are 1 MA of current at 1 MV in a 50-ns pulse. Parallel generators have reached levels of 10 TW.



**Figure 9.41** Schematic diagram of a pulsed power compression sequence.

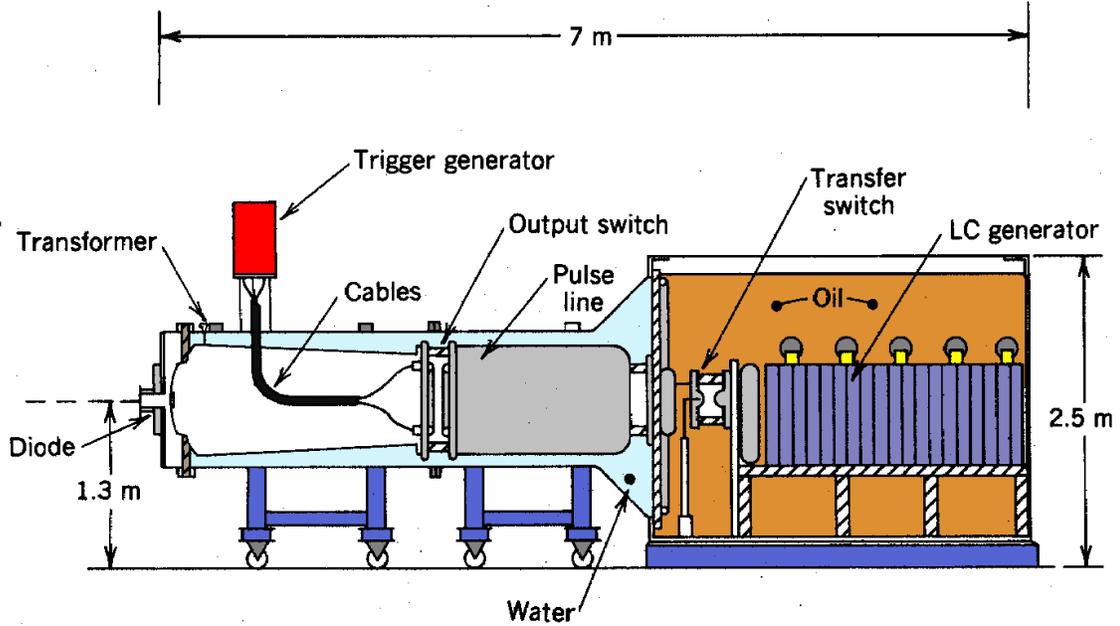
*Power compression* is the technique that has allowed the generation of such high output. A general power compression circuit is illustrated in Figure 9.41. Energy is transferred from one stage of energy storage to the next in an increasingly rapid sequence. Each storage stage has higher energy density and lower inductance. Even though energy is lost in each transfer, the decrease in transfer time is sufficient to raise the peak power.

The first power compression stage in Figure 9.41 is one we are already familiar with. A dc source charges an impulse generator such as a Marx generator. The output power from the Marx generator may be a factor of  $10^7$  higher than the average power from the source. In the example, the Marx generator transfers its energy to a low-inductance water-filled capacitor. A shorting switch then passes the energy to a low-impedance transmission line. A low-inductance multi-channel switch then connects the line to a vacuum load. Power multiplication in stages following the Marx generator are not so dramatic; gains become increasingly difficult. Figure 9.42 is a scale drawing of a pulsed power compression system to generate 300-kV, 300-kA, 80-ns pulses. The system consists of a low inductance 1-MV *LC* generator that pulse-charges a 1.5- $\Omega$  water-filled transmission line. Energy from the line is transferred by a multi-element high-pressure gas switch. The pulse is matched to a 1- $\Omega$  electron beam load by a coaxial line transformer.

The highest power levels have been achieved with multiple stages of capacitive energy storage connected by sequenced shorting switches. Triggered gas-filled switches are used in the early stages. Gas switches have too much inductance for later stages, so that self-breakdown between electrodes in a highly stressed liquid medium is used. Discharges in liquids absorb considerable energy. The resulting shock wave rapidly erodes electrodes. Therefore, machines of this type are fired typically 1-10 times per day and may need repair after 10-100 shots. In Section 9.13, we shall study a potential method for low-inductance switching at high repetition rate, saturable core magnetic switching. Characteristics of the EAGLE pulsed power generator are summarized in Table 9.4.

Transfer of energy between capacitors forms the basis of most pulsed power generators. A model for the transfer is illustrated in Figure 9.43. A capacitor is charged to voltage  $V_0$ , and then energy is switched through an inductance to a second capacitor by a shorting switch. The inductance may be

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**Figure 9.42** Neptune C, low-impedance pulsed power generator using overstressed water dielectric. (Courtesy H. Milde, Ion Physics Corporation.)

introduced purposely or may represent the inevitable parasitic inductance associated with current flow. Current in the circuit is described by the equation

$$-V_o + \int idt/C_1 + L (di/dt) + \int idt/C_2 = 0,$$

or

$$L (d^2i/dt^2) = -i (C_1+C_2)/C_1C_2. \quad (9.98)$$

If the switch, is closed at  $t = 0$ , then the initial conditions are  $i(0) = 0$  and  $di(0)/dt = V_o/L$ . The solution of Eq. (9.98) is

$$i(t) = V_o \sin(\omega t) / \sqrt{L(C_1+C_2)/C_1C_2}, \quad (9.99)$$

where

$$\omega = \sqrt{L C_1C_2/(C_1+C_2)}$$

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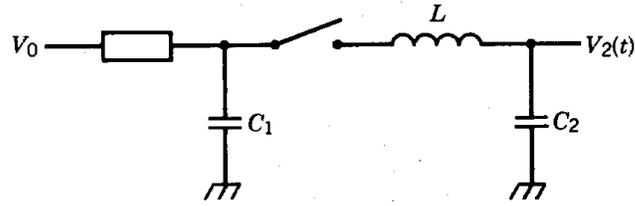
**TABLE 9.4 Parameters: EAGLE Pulsed Power Generator<sup>a</sup>**

EAGLE module		
1. Power compression cycle <sup>b</sup>		
Element	Energy Transfer Time ( $\mu$ s)	Output Switch
Marx generator	1.3	Gas switches (25), active trigger
Transfer capacitor water-filled	0.3	Triggered gas switch, high-voltage isolation
Charging pulseline	0.1	Multipin, self-breaking water switch
Pulse-forming line	0.1	Multipin, self-breaking water switch
2. Measured parameters		
Marx generator, stored energy		500 kJ
Marx generator voltage		3.15 MV
Energy transfer efficiency		50%
Output power		$4.5 \times 10^{12}$ W
Output energy		275 kJ
Output current (1.9- $\Omega$ load)		1.6 MA
Pulselength (FWHM power)		75 ns
Output voltage		3 MV
ROULETTE-X (Proposed)		
Function		Generation of Bremsstrahlung radiation and soft X rays for nuclear weapons effect simulation
Number of EAGLE modules		20
Design power at load		$7 \times 10^{13}$ W
Pulselength (FWHM power)		75 ns
Output voltage		2–2.5 MV
Output energy		5 MJ
Output current		20–22 MA

<sup>a</sup>Physics International Company

<sup>b</sup>Configuration: Marx generator with three stages of power compression using overstressed water dielectric. First two elements perform power compression in time, last two perform compression in space for multimodule compatibility.

### Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.43** Peaking capacitor circuit.

The quantities of interest are the time-dependent charge voltages on the two capacitors.

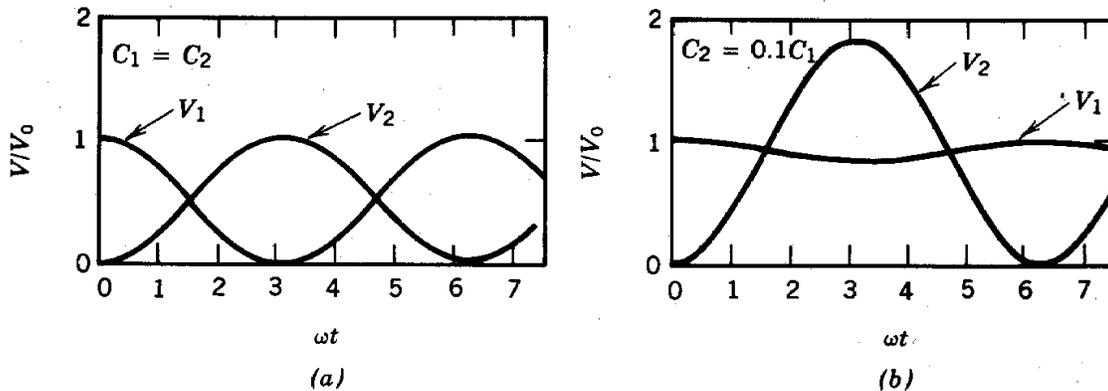
$$V_1(t) = V_o - \int idt/C_1, \quad V_2(t) = \int idt/C_2. \quad (9.100)$$

Substituting Eq. (9.99) into Eqs. (9.100), we find that

$$V_1 = V_o [1 - [C_2/(C_1+C_2)] (1 - \cos\omega t)], \quad (9.101)$$

$$V_2 = V_o [1 - [C_1/(C_1+C_2)] (1 - \cos\omega t)]. \quad (9.102)$$

Waveforms are plotted in Figure 9.44 for  $C_2 = C_1$  and  $C_2 \ll C_1$ . The first case is the optimum choice for a high-efficiency power compression circuit. A complete transfer of energy from the first to the second capacitor occurs at time  $t = \pi/\omega$ . In the second case, the energy transfer is inefficient but the second capacitor is driven to twice the charge voltage of the first. For this reason, the circuit of Figure 9.43 is often called the *peaking capacitor circuit*.



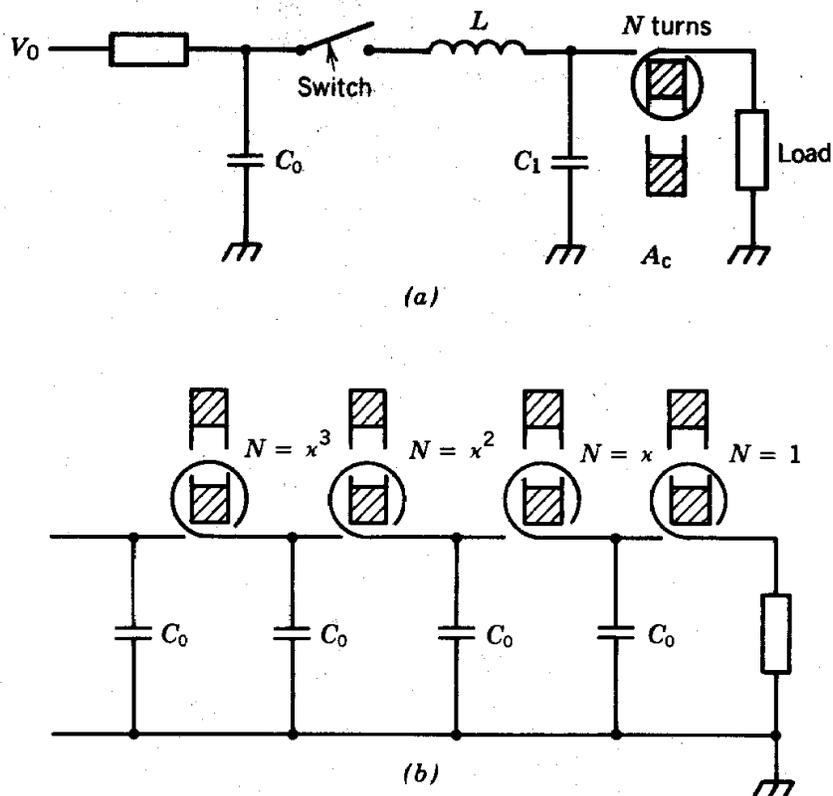
**Figure 9.44** Voltage waveforms on storage capacitor and charged capacitor in a peaking capacitor circuit. (a)  $C_2 = C_1$ . (b)  $C_2 = 0.1C_1$ .

### 9.13 PULSED POWER SWITCHING BY SATURABLE CORE INDUCTOR

Although the process of pulse shaping by saturable core ferromagnetic inductors has been used for many years in low-voltage circuits, application to high-power circuits is a recent area of interest. Magnetic switches can be constructed with low inductance and high transfer efficiency. Energy losses are distributed evenly over the core mass, and there is no deionization time as there is in gas or liquid breakdown switches. These factors give magnetic switches potential capability for high repetition rate operation.

A two-stage power compression circuit with a magnetic switch is shown in Figure 9.45a. Capacitors  $C_0$  and  $C_1$  constitute a peaking circuit. Energy is transferred by a normal shorting switch. The energy in  $C_1$  is then routed to a load by a saturable core inductor switch. The circuit achieves power compression if the switch out time from  $C_1$  is short compared to the initial transfer. We choose  $C_0 = C_1$  for high efficiency.

In order to understand the operation of the circuit of Figure 9.45 a, we must refer to the hysteresis curve of Figure 5.12. We assume that the ferromagnetic core of the inductor is ideal; its properties are described by a static hysteresis curve. This means that there are no real currents



**Figure 9.45** Saturable core magnetic switching. (a) Peaking capacitor circuit with magnetic output switch. (b) Multistage power compression circuit with uniform capacitors and ferromagnetic cores.

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flowing in the core material; magnetic effects are produced solely by alignment of atomic currents. Methods of core construction to assure this condition are described in Chapter 10. We further assume that at some time before  $t = 0$  a separate circuit has pulsed a negative current through the ferromagnetic core sufficient to bias it to  $-B_s$ . After the current is turned off, the core settles to a state with flux level  $-B_r$ . This process is called *core reset*.

Consider the sequence of events after the shorting switch is activated. At early times, the right-hand portion of the circuit is approximately an open circuit because of the high inductance of the winding around the high  $\mu$  core. Energy flows from  $C_0$  to  $C_1$ ; the voltage on  $C_1$  is given by Eq. (9.102). Further, Faraday's law implies that

$$V_1(t) = N A_c (dB/dt), \quad (9.103)$$

where  $A_c$  is the cross-section area of the core and  $N$  is the number of windings in the inductor. The core reaches saturation when

$$\int V_1(t) dt = N A_c (B_s + B_r). \quad (9.104)$$

After saturation, the inductance  $L_2$  decreases by a large factor, approaching the vacuum inductance of the winding. The transition from high to low inductance is a bootstrapping process that occurs rapidly. Originally, translation along the H axis of the hysteresis curve is slow because of the high inductance. Near saturation, the inductance drops and the rate of change of leakage current increases. The rate of change of H increases, causing a further drop in the inductance. The impedance change of the output switch may be as fast as 5 ns for a well-designed core and low-inductance output winding.

Energy is utilized efficiently if the output switch has high impedance for  $0 \leq t \leq \pi/\omega$  and the switch core reaches saturation at the end of the interval when all the circuit energy is stored in  $C_1$ . Integrating Eq. (9.103) over this time span gives the following prescription for optimum core parameters

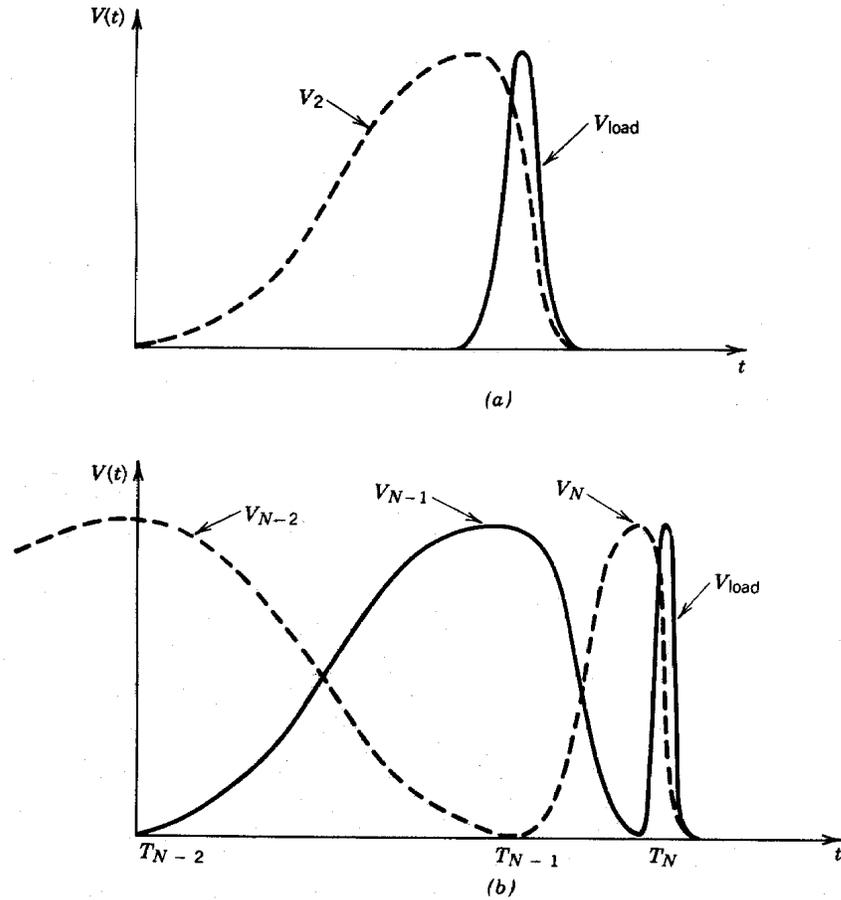
$$N A_c (B_s + B_r) = V_o \pi / 2\omega. \quad (9.105)$$

Power compression in the two-stage circuit is, illustrated in Figure 9.46a.

A multiple compression stage circuit with magnetic switching is shown in Figure 9.45b. The parameters of the saturable cores are chosen to transfer energy from one capacitor to the next at increasing power levels. Current waveforms in the switches are shown in Figure 9.46b. There are a number of constraints on power compression circuits:

1. The capacitance in all stages should be equal to  $C_0$  for high efficiency.
2. Switching of each stage should occur at peak energy transfer. This means that

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.46** Voltage waveforms—saturable core magnetic switching. (a) Voltage on charged capacitor and load in single-stage circuit. (b) Voltages on last three capacitors and load in multistage network.

$$V_0 \pi \sqrt{L_{n-1}^s C_0} / 2 = 2N_n A_c (B_s + B_r). \quad (9.106)$$

In Eq. (9.106),  $L_{n-1}^s$  is the saturated (or vacuum) inductance of the winding around core  $n-1$  and  $N_n$  is the number of turns around core  $n$ .

3. The unsaturated inductance of winding  $n$  should be much larger than the saturated inductance of winding  $n-1$ , or

$$L_n^u \gg L_{n-1}^s. \quad (9.107)$$

Equation (9.107) implies that prepulse is small; power does not move forward in the circuit until

## Electrostatic Accelerators and Pulsed High Voltage

switching time.

4. The saturated inductance of winding  $n$  should be small compared to the saturated inductance of winding  $n-1$ ,

$$L_n^u \ll L_{n-1}^u. \quad (9.108)$$

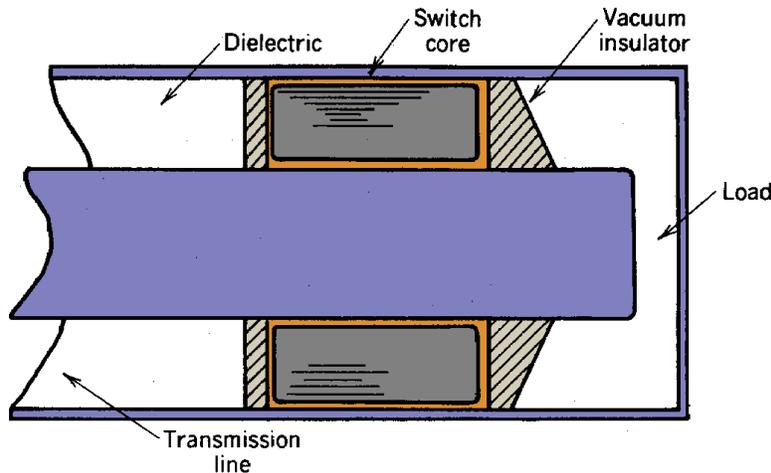
Equation (9.108) guarantees that the time for energy transfer from  $C_n$  to  $C_{n+1}$  is short compared to the time for energy to flow backward from  $C_n$  to  $C_{n-1}$ ; therefore, most of the energy in the circuit moves forward.

One practical way to construct a multistage power compression circuit is to utilize identical cores (constant  $A_c$ ,  $B_s$  and  $B_r$ ) and capacitors (constant  $C_0$ ) and to vary the number of turns around each core to meet the conditions listed above. In this case, the first and second conditions imply that the number of turns decreases geometrically along the circuit or  $N_n = B_{n-1}/\kappa$ . If we assume that the saturated and unsaturated inductances vary by a factor  $\mu/\mu_o$ , conditions 3 and 4 imply that  $\kappa^2 \ll (\mu/\mu_o)$  and  $\kappa^2 \gg 1$ . These conditions are satisfied if  $1 < \kappa < \sqrt{\mu/\mu_o}$ . The power compression that can be attained per stage is limited. The time to transfer energy scales as

$$\tau_n/\tau_{n-1} \cong \sqrt{L_n/L_{n-1}} \cong 1/\kappa. \quad (9.109)$$

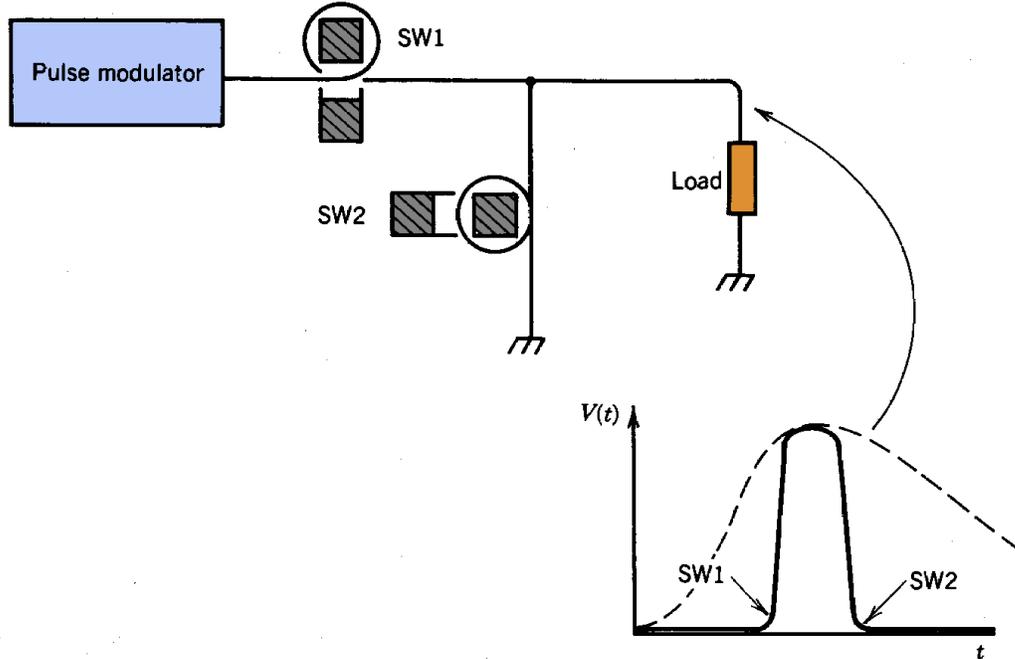
The power compression ratio is the inverse of Eq. (9.109). For ferrite cores with  $\mu/\mu_o \cong 400$ , the maximum power compression ratio per stage is about 5.

Magnetic switches in low-power circuits are usually toroidal pulse cores with insulated wire



**Figure 9.47** Pulse-charged coaxial transmission line with a low-inductance, saturable core magnetic output switch.

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**Figure 9.48** Pulse shaping with saturable core magnetic switches.

windings. Magnetic switches used to control the output of a high-power pulse-charged transmission line must be designed for high-voltage standoff and low inductance. A typical configuration is shown in Figure 9.47. Saturable core inductors can also be used for pulselength shortening and risetime sharpening if efficiency is not a prime concern. The circuit illustrated in Figure 9.48 produces a short, fast-rising voltage pulse from a slow pulse generator.

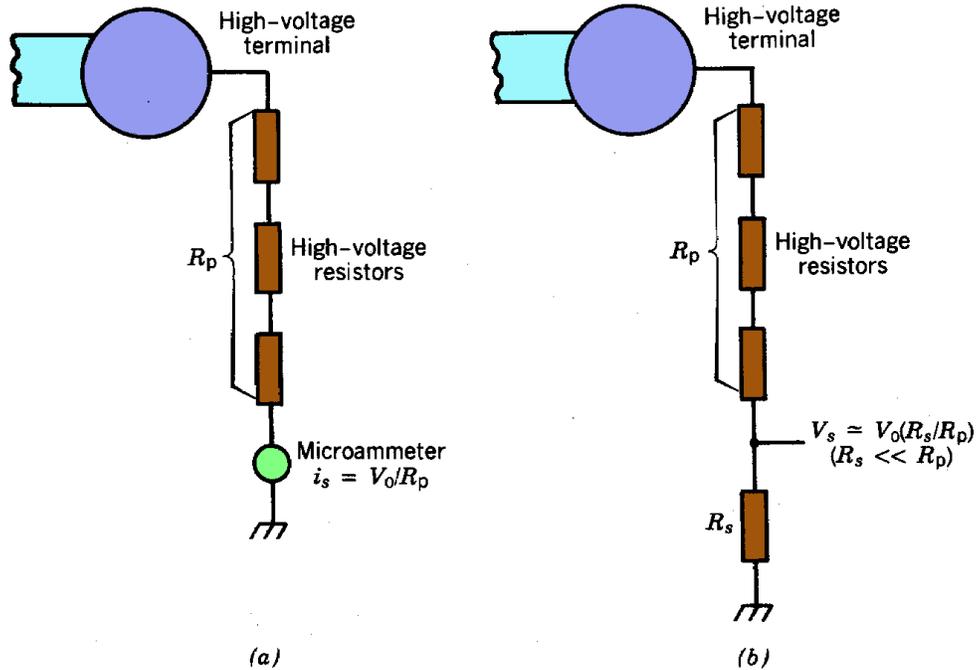
### 9.14 DIAGNOSTICS FOR PULSED VOLTAGES AND CURRENTS

Accurate measurements of acceleration voltage and beam current are essential in applications to charged particle acceleration. The problem is difficult for pulsed beams because the frequency response of the diagnostic must be taken into account. In this section, we shall consider some basic diagnostic methods.

The diagnostic devices we shall discuss respond to electric or magnetic fields over a limited region of space. For instance, measurement of the current through a resistor gives the instantaneous spatially-averaged electric field over the dimension of the resistor. Confusion often arises when electric field measurements are used to infer a voltage. In particle acceleration applications voltage usually means the energy a charged particle gains or loses crossing the region of measurement. Signals from voltage diagnostics must be carefully interpreted at high frequency because of two effects.

1. Particle energy gain is not equal to  $\int \mathbf{E} \cdot d\mathbf{x}$  (where the integral is taken at a fixed time) when the

## Electrostatic Accelerators and Pulsed High Voltage



**Figure 9.49** High-voltage measurements with resistor strings. (a) Resistive shunt. (b) Resistive divider.

time interval for the particle to cross the region is comparable to or less than the period of voltage oscillations. This is called the *transit time effect* and is treated in Section 14.4.

2. The voltage difference between two points is not a useful concept when the wavelength of electromagnetic oscillations is less than or comparable to the dimension of the region. In this case, electric field may vary over the length of the diagnostic. For example, in the presence of a bipolar electric field pattern, a voltage diagnostic may generate no signal.

Similarly, magnetic field measurements can be used to infer current flow at low frequency, but caution should be exercised in interpretation at high frequency.

### A. Steady-State Voltage Measurement

The only, time-independent device that can measure a dc voltage is the resistive shunt (Fig. 9.49a). The resistive shunt consists of a resistor string of value  $R_p$  attached between the measurement point and ground. The quantity  $R_p$  must be high enough to prevent overloading the voltage source or overheating the resistors. The net voltage can be inferred by measuring current flow through the chain with a microammeter,

$$i_s = V_0/R_p. \tag{9.110}$$

## Electrostatic Accelerators and Pulsed High Voltage

Another approach is to measure the voltage across a resistor  $R_s$  at the low-voltage end of the chain. This configuration is called a *resistive divider* (Fig. 9.49b). The division ratio, or the signal voltage divided by the measured voltage, is

$$V_s/V_0 = R_s / (R_p + R_s). \quad (9.111)$$

Capacitive voltage measurements of dc voltages are possible if there is a time variation of capacitance between the source and the diagnostic. The charge stored in a capacitance between a high-voltage electrode and a diagnostic plate ( $C_p$ ) is

$$Q = C_p V_0.$$

The total time derivative of the charge is

$$i_s = dQ/dt = C_p (dV_0/dt) + V_0 (dC_p/dt). \quad (9.112)$$

In pulsed voltage measurements,  $C_p$  is maintained constant and a diagnostic current is generated by  $dV_0/dt$ . Measurements of dc voltages can be performed by varying the capacitance as indicated by the second term on the right-hand side of Eq. (9.112).

The voltage probe of Figure 9.50 has an electrode exposed to electric fields from a high-voltage terminal. A rotating grounded disc with a window changes the mutual capacitance between the probe and high-voltage electrodes, inducing a current. The following example of a voltage measurement on a Van de Graaff accelerator illustrates the magnitude of the signal current. Assume the high-voltage electrode is a sphere of radius  $R_o$  in a grounded spherical chamber of radius  $R_I$ . With gas insulation, the total capacitance between the outer and inner electrodes is

$$C_t \cong 4\pi\epsilon_o R_o R_I / (R_I - R_o). \quad (9.113)$$

We assume further that the probe electrode and rotating window are near ground potential and that their surfaces are almost flush with the surface of the outer electrode. Field lines between the high-voltage electrode and the probe are radial; therefore, the mutual capacitance is equal to  $C_t$  times the fraction of outer electrode area occupied by the probe. If  $A_p$  is the area of the hole in the rotating plate, then the capacitance between the high-voltage electrode and probe varies between

$$0 \leq C_p \leq C_t A_p / 4\pi R_I^2.$$

As an example, take  $V_0 = 1$  MV,  $R_o = 0.3$  m,  $R_I = 1$  m, and  $A_p = 8 \times 10^{-3}$  m<sup>2</sup>. A plate, rotating at 1000 rpm ( $\omega = 100$  s<sup>-1</sup>) obscures the probe half the time. The total spherical capacitance is  $C_t =$

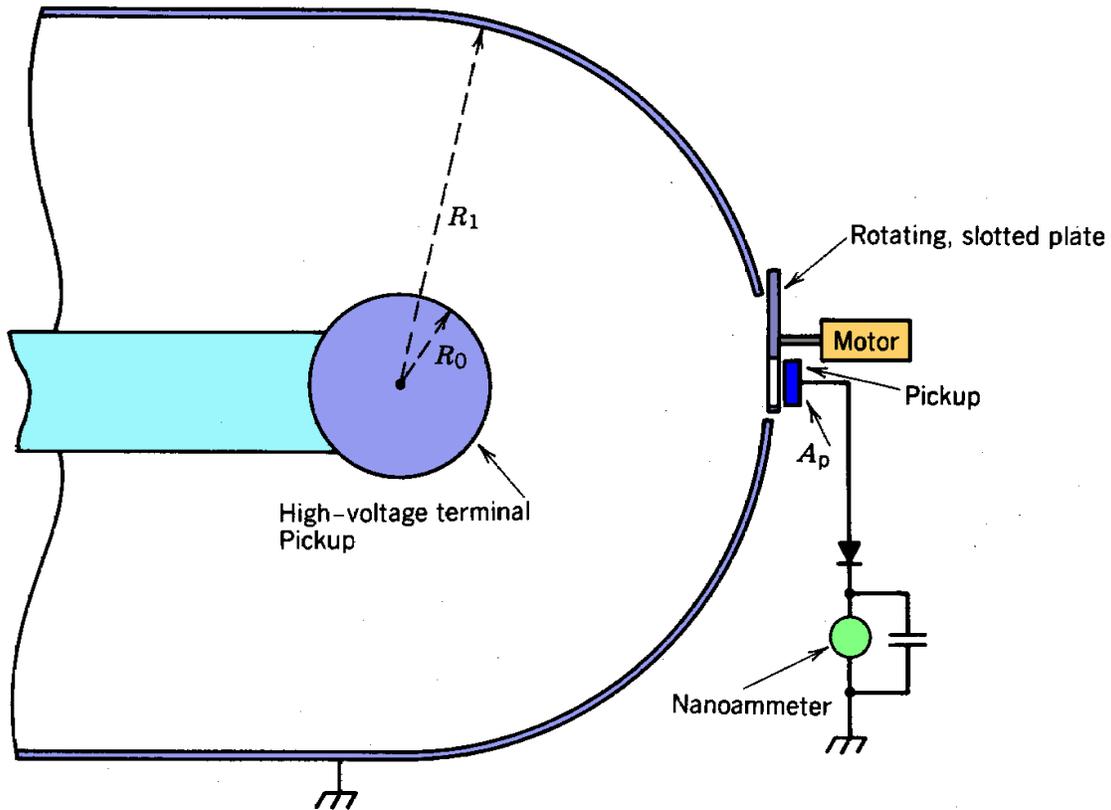


Figure 9.50 Generating voltmeter; high-voltage probe with time-varying capacitance.

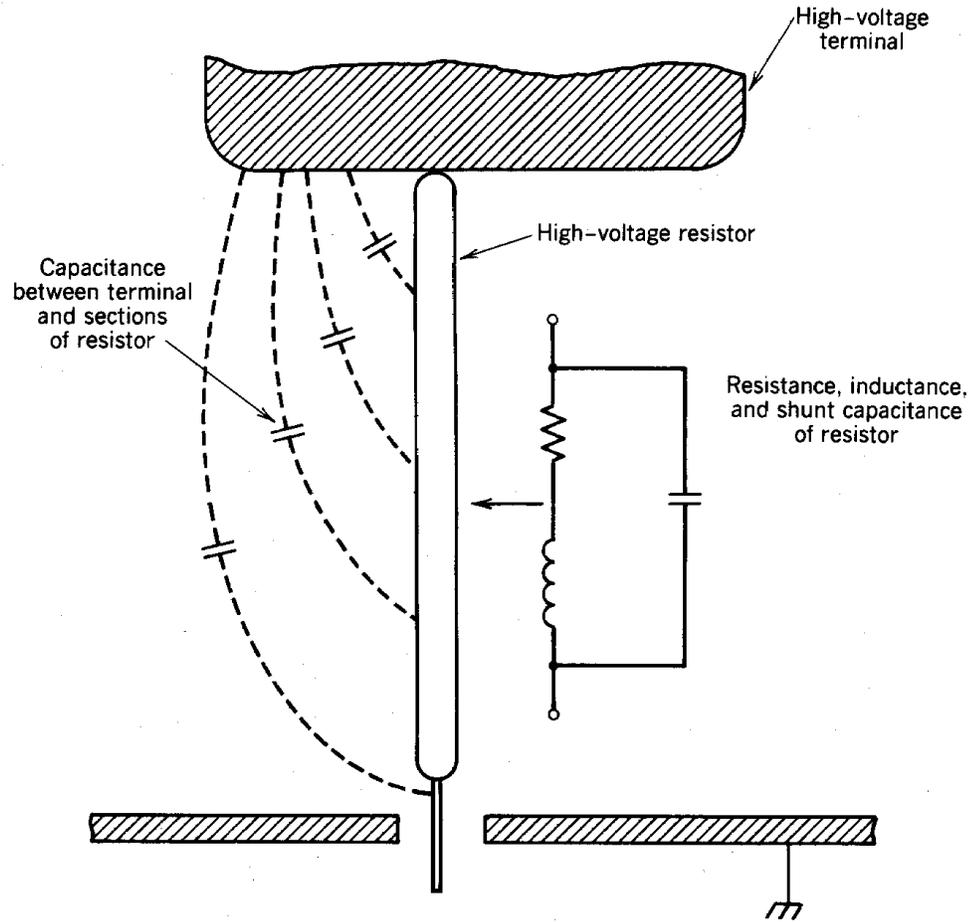
100 pF; the mutual capacitance between terminal and probe is 0.07 pF. The signal current is approximately  $i_s \approx V_0 (\omega C_p/2) \cos \omega t$ . Substituting values, the magnitude of the ac current is 3.5  $\mu$ A, an easily measured quantity.

### B. Resistive Dividers for Pulsed Voltages

The electrostatic approximation is usually applicable to the measurements of output voltage for pulsed power modulators. For example, electromagnetic waves travel 15 m in vacuum during a 50-ns pulse. The interpretation of voltage monitor outputs is straightforward as long as the dimensions of the load and leads are small compared to this distance.

Resistive dividers are well suited to pulsed voltage measurements, but some care must be exercised to compensate for frequency-dependent, effects. Consider the divider illustrated in Figure 9.49. Only the resistance values are important for dc measurements, but we must include effects of capacitance and inductance in the structure for pulsed measurements. A more detailed circuit model for the resistive divider is shown in Figure 9.51. There is inductance associated with current flow through the resistors and a shunt capacitance between points on the divider.

## Electrostatic Accelerators and Pulsed High Voltage



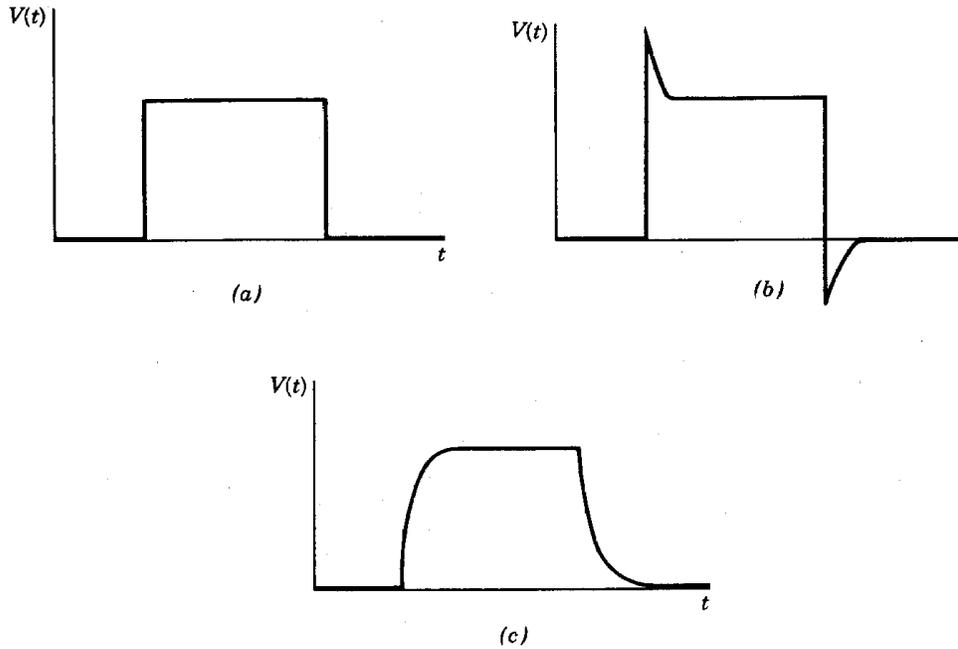
**Figure 9.51** Effect of stray capacitance, shunt capacitance, and series inductance on resistive voltage divider.

A particularly unfavorable situation occurs when the primary resistor chain is a water solution resistor and the signal resistor is an ordinary carbon-composition resistor. Water resistors have high dielectric strength and good energy-absorbing ability, but they also have a large shunt capacitance,  $C_p$ . Therefore, on time scales less than  $R_0 C_p$ , the circuit acts as a differentiator. A square input pulse gives the pulse shape shown in Figure 9.52b.

The above example illustrates a general problem of pulsed voltage attenuators; under some circumstances, they may not produce a true replication of the input waveform. We can understand the problem by considering the diagnostic as a device that transforms an input waveform to an output waveform. We can represent the process mathematically by expressing the input signal as a Fourier integral,

$$V_0(t) = \int a(\omega) \exp(j\omega t), \quad (9.114)$$

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**Figure 9.52** Waveforms from resistive voltage divider. (a) Ideal output voltage from a square-pulse input. (b) Output with significant shunt capacitance. (c) Output with significant series inductance.

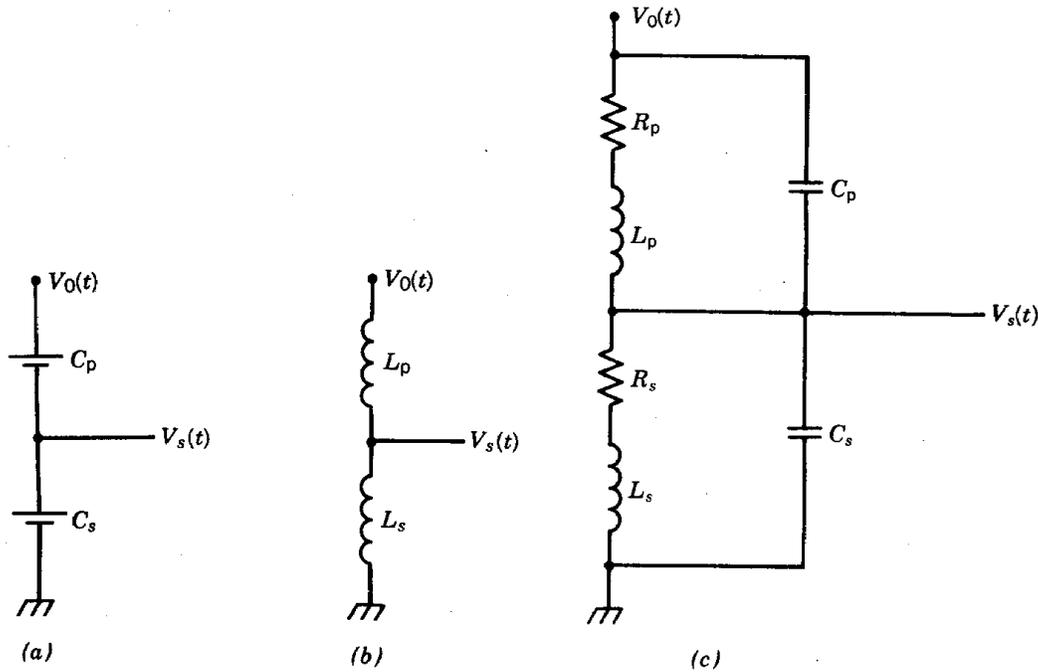
where  $a(\omega)$  is a complex number. Neglecting the electromagnetic transit time through the diagnostic, the transform function of a diagnostic system is a function of frequency,  $g(\omega)$ . The quantity  $g(\omega)$  is a complex number, representing changes in amplitude and phase. The output signal is

$$V_s(t) = \int a(\omega) g(\omega) \exp(j\omega t). \quad (9.115)$$

In an ideal diagnostic, the transformation function is a real number independent of frequency over the range of interest. When this condition does not hold, the shape of the output signal differs from the input. In the case of the water solution resistive divider,  $g(\omega)$  is constant at low frequency but increases in magnitude at high frequency. The result is the spiked appearance of Figure 9.52b. Conversely, if the inductance of the divider stack is a dominant factor, high frequencies are inhibited. The pulse risetime is limited to about  $L/R_0$ , producing the pulse shown in Figure 9.52c.

Pulse shapes can often be reconstructed by computer if the transfer function for a diagnostic and the associated cabling is known. Nonetheless, the best practice is to design the diagnostic for flat frequency response. Devices with frequency variations must be compensated. For example, consider the water solution resistive divider. A method of compensation is evident if we recognize that pulsed voltage dividers can be constructed with capacitors or inductors. In the capacitive divider (Fig. 9.53a), the division ratio is

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**Figure 9.53** Voltage dividers. (a) Capacitive. (b) Inductive. (c) Balanced voltage divider.

$$V_s/V_0 = C_p/(C_s + C_p), \quad (9.116)$$

and for an inductive divider (Fig. 9.53b),

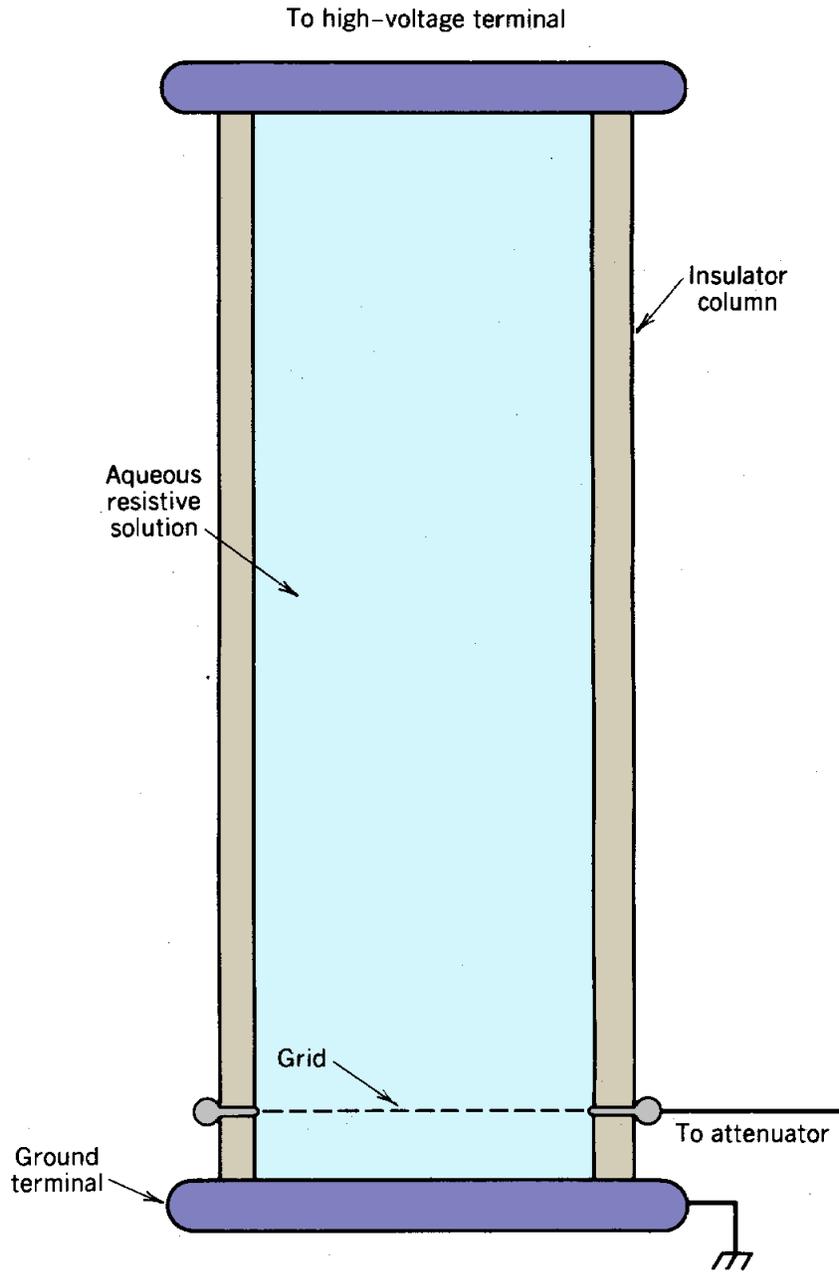
$$V_s/V_0 = L_s/(L_s + L_p), \quad (9.117)$$

A balanced divider is illustrated in Figure 9.53c. Extra circuit components ( $C_s$  and  $L_s$ ) have been added to the probe of Figure 9.51 to compensate for the capacitance and inductance of the probe. The divider is balanced if

$$R_s/(R_s + R_p) = C_p/(C_s + C_p) = L_s/(L_s + L_p). \quad (9.118)$$

The component  $C_s$  pulls down the high-frequency components passed by  $C_p$  eliminating the overshoot of Figure 9.52a. Similarly, the inductance  $L_s$  boosts low-frequency components which were over-attenuated by  $L_p$  to improve the risetime of Figure 9.52b. Sometimes,  $C_p$  and  $L_p$  are not known exactly, so that variable  $C$  and  $L$  are incorporated. These components are adjusted to give the best output for a fast-rising square input pulse.

Divider balance can also be achieved through geometric symmetry. A balanced water solution resistive divider is shown in Figure 9.54. The signal pick-off point is a screen near the ground plane. The probe resistor and signal resistor,  $R_p$  and  $R_s$ , share the same solution. The division ratio

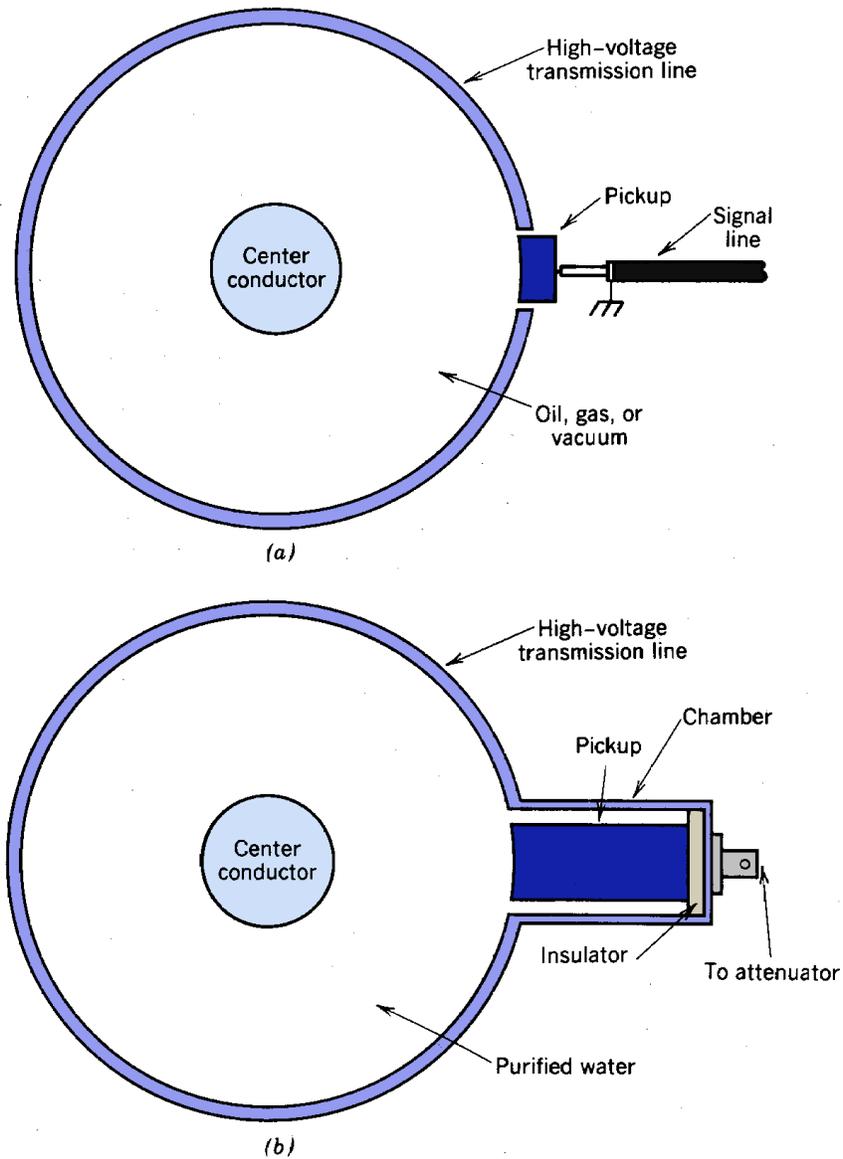


**Figure 9.54** Balanced voltage divider with water solution resistor.

is given by the ratio of the length of  $R_s$  to the length of  $R_p$ . The high dielectric constant of the water solution assures that electric field lines in the water are parallel to the column [see Eq. (5.6)]. Inspection of Figure 9.54 shows that the resistances, capacitances, and inductances are all in the proper ratio for a balanced divider when the solution resistance, resistor diameter, and electric field are uniform along the length of the resistor.

C. Capacitive Dividers for Pulsed Voltages

High-voltage dividers can be designed with predominantly capacitive coupling to the high-voltage electrode. Capacitive probes have the advantage that there is no direct connection and there are negligible electric field perturbations. Two probes for voltage measurements in high-voltage transmission lines are shown in Figures 9.55a and b.



**Figure 9.55** Capacitive voltage monitors in high-voltage transmission line. (a) Capacitive  $dV/dt$  pickup. (b) Balanced capacitive divider.

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The first probe is suitable for use in an oil or gas-filled line where the resistance of the medium is effectively infinite and the mutual capacitance between terminal and probe is low. The capacitance between the probe and the wall,  $C_s$ , is purposely made small. The probe is connected directly to a signal transmission line of impedance  $Z_o$ . The signal voltage on the line is

$$V_s \approx (C_p Z_o) (dV_0/dt). \quad (9.119)$$

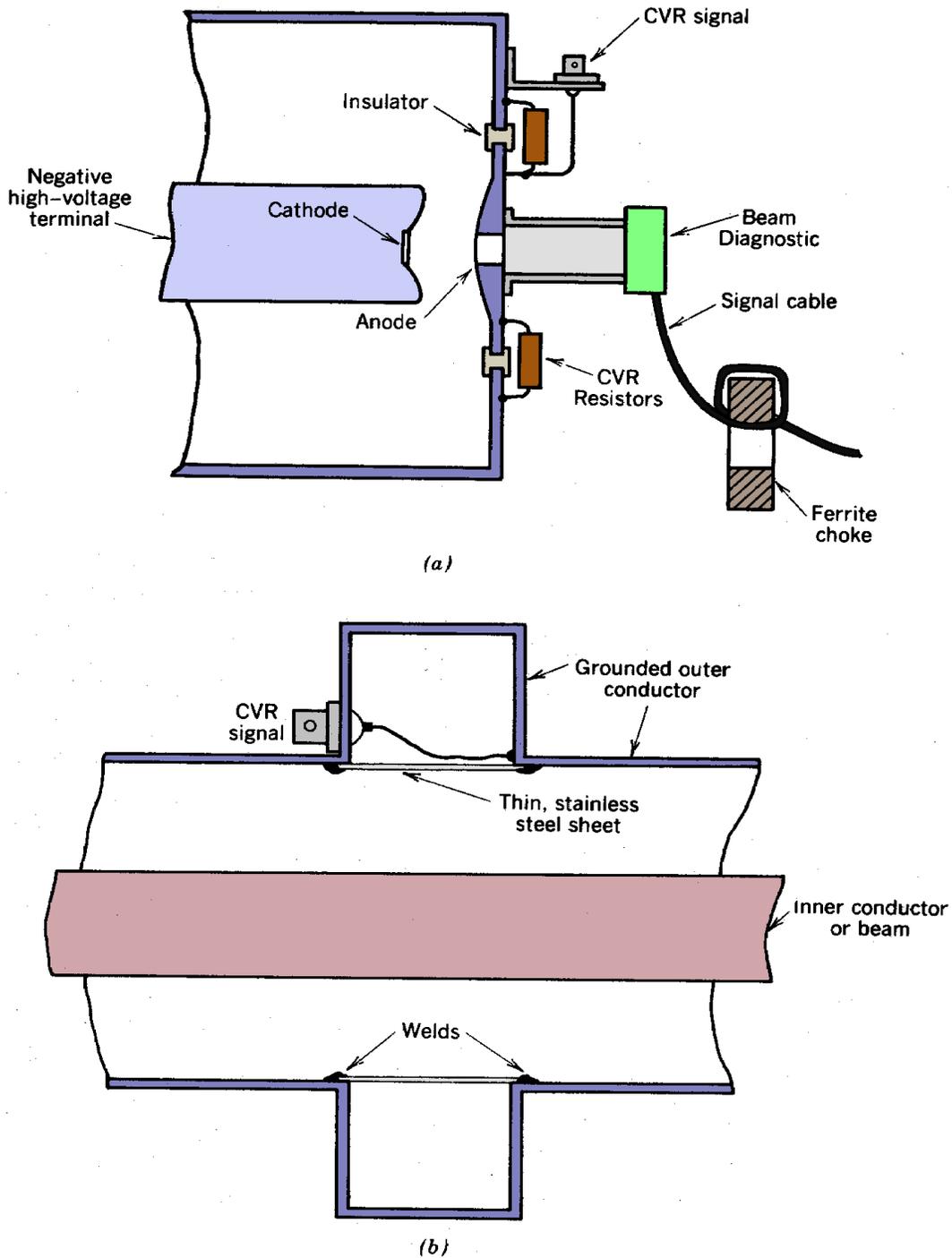
This signal is processed by an integration circuit (Section 9.1). The minimum time response of the probe is about  $Z_o C_s$ . The maximum time is determined by the integration circuit.

The second probe is a self-integrating probe for use in water-filled transmission lines. The coupling capacitance is high in water lines; in addition, there is real current flow because of the non-zero conductance. In this situation, the capacitance between the probe and the wall is made high by locating the probe in a reentrant chamber. The probe acts as a capacitive divider rather than a coupling capacitor. If the space between the probe electrode and the wall of the chamber is filled with a non-conductive dielectric, the divider is unbalanced because of the water resistance. Balance can be achieved if the water of the transmission line is used as the dielectric in the probe chamber. The minimum time response of the divider is set by electromagnetic pulse transit time effects in the probe chamber. Low-frequency response is determined by the input resistance of the circuit attached to the probe. If  $R_d$  is the resistance, then the signal of a square input pulse will droop as  $\exp(-t/R_d C_s)$ . Usually, a high series  $R_d$  is inserted between the transmission line and the probe to minimize droop and further attenuate the signal voltage.

### D. Pulsed Current Measurements with Series Resistors

Measurements of fast-pulsed beam flux or currents in modulators are usually performed with either series resistors, magnetic pickup loops, or Rogowski loops. A typical series resistor configuration for measuring the current from a pulsed electron gun is shown in Figure 9.56a. Power is supplied from a voltage generator through a coaxial transmission line. The anode is connected to the return conductor through a series resistor called a *current-viewing resistor* (CVR). Current is inferred from the voltage across the resistor. The CVR has low resistance,  $R_s$ , and very low series inductance. In many cases, diagnostic cables and power leads are connected to the anode. In this situation, some circuit current flows through the ground connectors of the cables, ultimately returning to the generator. This current does not contribute to a signal on the CVR. The current loss is minimized if the inductance associated with the loss paths is large compared to the series inductance of the CVR. When this condition holds, most of the primary circuit current flows through the CVR for times short compared to  $L/R_s$ , where  $L$  is the inductance along the connecting cables.

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**Figure 9.56** Current-viewing resistors. (a) Typical arrangement for current measurements from a pulsed electron injector with inductive isolation for beam diagnostics. (b) Low-inductance CVR for high-power transmission line or beamline.

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A design used for commercial CVRs is shown in Figure 9.56b. The diagnostic is mounted on the return conductor of a coaxial transmission line. It consists of a toroidal chamber with a thin sheet of stainless steel welded across the opening to preserve the cylindrical geometry. In the absence of the resistive sheet, current flow generates a magnetic field inside the chamber; there is an inductance  $L_s$ , associated with the convoluted path of return current. When a sheet with resistance  $R_s$  is added, current flows mainly through the sheet for times less than  $L_s/R_s$ . When this condition holds, voltage measured at the pick-off point is approximately equal to  $iR_s$ . The response of the CVR can be extended to lower frequencies by the inclusion of a ferromagnetic material such as ferrite in the toroidal chamber. The principle of operation of the ferrite-filled probe will be clear when we study inductive linear accelerators in the next chapter.

### E. Pulsed Current Measurements with Magnetic Pickup Loops

The magnetic pickup loop measures the magnetic field associated with current flow. It consists of a loop normal to the magnetic field of area  $A$  with  $N$  turns. If the signal from the loop is passed through a passive integrator with time constant  $RC$ , then the magnetic field is related to the integrator output voltage by

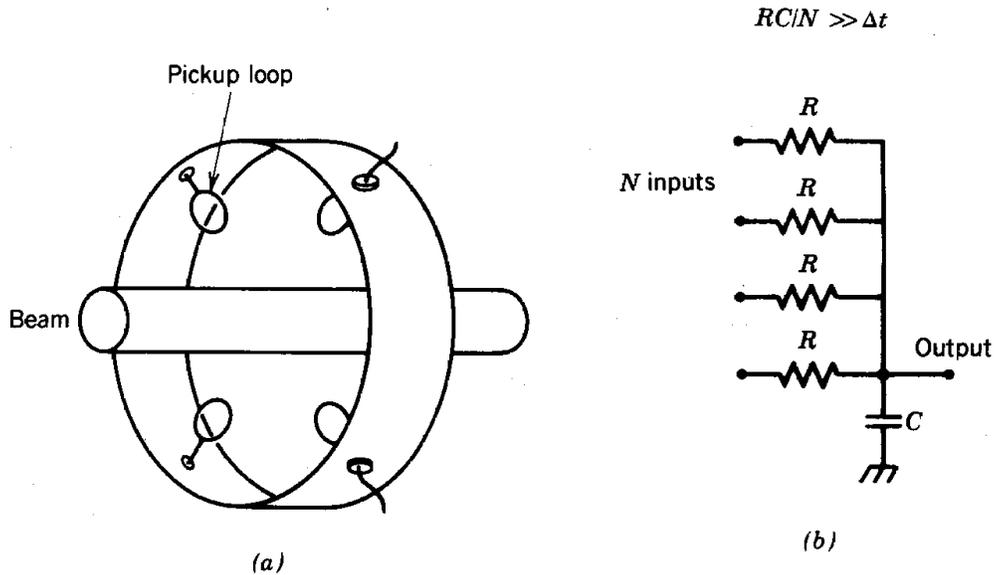
$$V_s = NAB/RC. \quad (9.120)$$

Current can be inferred if the geometry is known. For instance, if the current is carried by the center conductor of a transmission line or a cylindrically symmetric beam on axis, the magnetic field at a probe located a distance  $r$  from the axis is given by Eq. (4.40). The loop is oriented to sense toroidal magnetic fields.

The net current of a beam can be determined even if the beam moves off axis by adding the signals of a number of loops at different azimuthal positions. Figure 9.57 shows a simple circuit to add and integrate the signals from a number of loops. Magnetic pickup loops at diametrically opposite positions can detect beam motion off-axis along the diameter. In this case, the loop signals are subtracted to determine the difference signal. This is accomplished by rotating one of the loops  $180^\circ$  and adding the signals in the circuit of Figure 9.57.

The high-frequency response of a magnetic pickup loop is determined by the time it takes magnetic fields to fill the loop. The loop is diamagnetic; currents induced by changing flux produce a counterflux. The response time for a loop with inductance  $L$  and a series resistance  $R$  is  $L/R$ . The resistance is usually the input impedance of a transmission line connected to a loop. Sensitive magnetic pickup loops with many turns generally have slow time response.

The magnetic pickup loop has a useful application in pulsed -voltage systems when the voltage diagnostic cannot be attached at the load. For instance, in driving a high-current electron extractor with a transmission line modulator, it may be necessary to measure voltage in the line rather than at the vacuum load. If there is inductance  $L$  between the measurement point and the acceleration gap,



**Figure 9.57** (a) Multiple magnetic pickup loops for measurements of net beam current or beam position. (b) Passive integrator with provision for adding (or subtracting) probe signals.

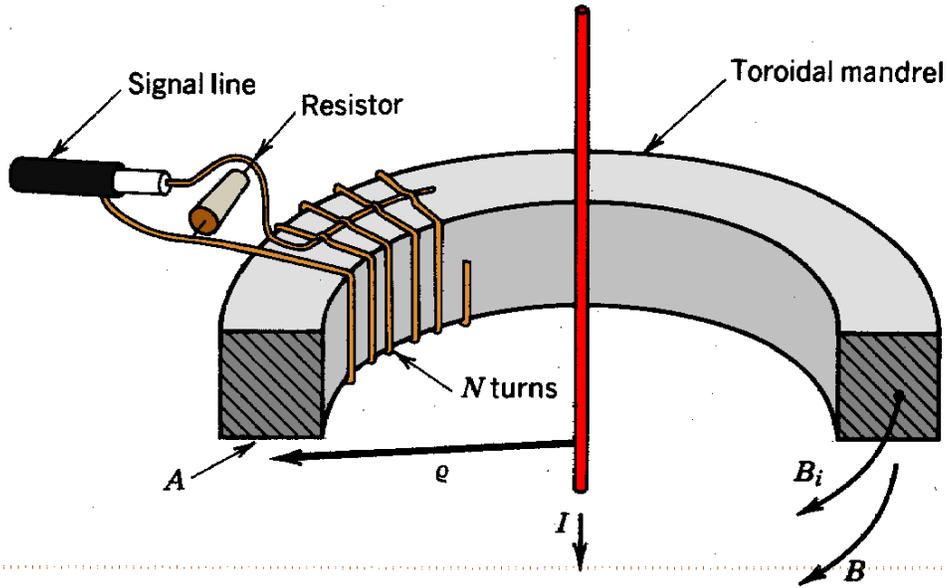
the actual voltage on the gap is

$$V_g(t) = V_0(t) - L_p (di/dt). \tag{9.121}$$

The quantity  $V_0$  is the voltage at the measurement point and  $V_g$  is, the desired voltage. A useful measurement can often be achieved by adding an inductive correction to the signal voltage. An unintegrated magnetic pickup loop signal (proportional to  $-di/dt$ ) is added to the uncorrected voltage through a variable attenuator. The attenuator is adjusted for zero signal when the modulator drives a shorted load. The technique is not reliable for time scales comparable to or less than the electromagnetic transit time between the measurement point and the load. A transmission line analysis must be used to infer high-frequency correction for the voltage signal.

### F. Rogowski Loop

The final current diagnostic we will study is the Rogowski loop (Fig. 9.58). It is a multi-turn toroidal inductor that encircles the current to be measured. When the dimension of the loops is small compared to the major radius, Eq. (4.39) implies that the net flux linking the series of loops does not depend on the position of the measured current inside the loop. Adopting this approximation, we assume the magnetic field is constant over the cross-sectional area of the windings and approximate the windings as a straight solenoid.



**Figure 9.58** Rogowski loop—self-integrating current probe.

The quantity  $B(t)$  is the magnetic field produced by the measured current and  $B_i(t)$  is the total magnetic field inside the loop. The loop has  $N$  windings and a series resistance  $R$  at the output. The current flowing in the loop circuit is the total loop voltage divided by  $R$  or

$$i(t) = \frac{NA (dB_i/dt)}{R} \quad (9.122)$$

The field inside the loop is the applied field minus the diamagnetic field associated with the loop current or

$$B_i(t) = B(t) - \mu_o i(t) (N/2\pi r). \quad (9.123)$$

Substituting Eq. (9.122) into (9.123),

$$(\mu_o N^2 A / 2\pi r R) (dB_i/dt) + B_i = (L/R) (dB_i/dt) + B_i = B(t). \quad (9.124)$$

The second form is derived by substituting the expression for the inductance of a toroidal winding, Eq. (9.18). In the limit that the time scale for variations of the measured current  $I$  is short compared to  $L/R$ , the first term on the left-hand side is large compared to the second term. In this limit very little of the magnetic field generated by the current to be measured penetrates into the winding. Dropping the second term and substituting Eqs. (9.122) and (4.40), we find that

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$$i = I/N, \quad (9.125)$$

where  $i$  is the loop output current and  $I$  is the measured current.

In summary, the Rogowski loop has the following properties:

1. The output signal is unaffected by the distribution of current encircled by the loop.
2. The Rogowski loop is a self-integrating current monitor for time scales  $\Delta t \ll L/R$ .
3. The probe can respond to high-frequency variations of current.

The low-frequency response of the Rogowski loop can be extended by increasing the winding inductance at the expense of reduced signal. The inductance is increased a factor  $\mu/\mu_0$  by winding the loop on a ferrite torus. In this case, Eq. (9.125) becomes

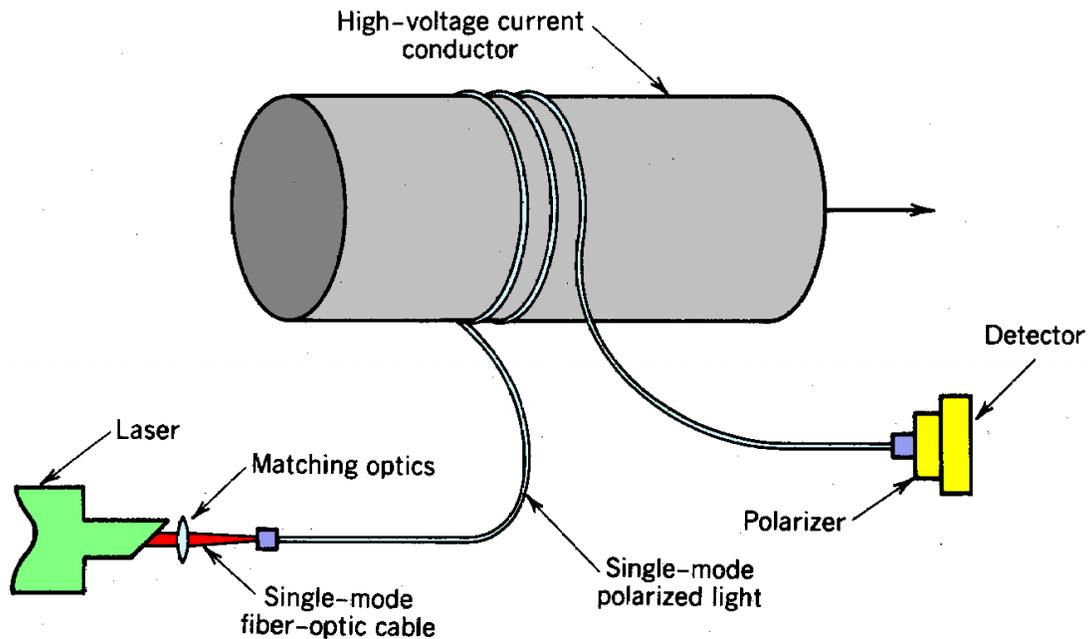
$$i = \frac{I}{N (\mu/\mu_0)}. \quad (9.126)$$

### G. Electro-optical Diagnostics

The diagnostics we have considered are basic devices that are incorporated on almost all pulsed voltage systems. There has been considerable recent interest in the use, of electro-optical techniques to measure rapidly varying electric and magnetic fields. The main reason is the increasing, use of digital data acquisition systems. Optical connections isolate sensitive computers from electromagnetic noise, a particular problem for pulsed voltage systems.

A diagnostic for measuring magnetic fields is illustrated in Figure 9.59. Linearly polarized light from a laser is directed through a single-mode, fiber-optic cable. The linearly polarized light wave can be resolved into two circularly polarized waves. A magnetic field parallel to the cable affects the rotational motion of electrons in the cable medium. The consequence is that the index of refraction for right and left circularly polarized waves is different. There is a phase difference between the waves at the end of the cable. The phase difference is proportional to the strength of the magnetic field integrated along the length of the cable,  $\int \mathbf{B} \cdot d\mathbf{l}$ . When the waves are recombined, the direction of the resulting linearly polarized wave is rotated with respect to the initial wave. This effect is called *Faraday rotation*. The technique can be used for current diagnostics by winding one or more turns of the fiber-optic cable around the measured current path. Equation (4.39) implies that the rotation angle is proportional to the current. Electric fields can be sensed through the Kerr effect or the Pockels effect in an optical medium. Interferometric techniques are used to measure changes in the index of refraction of the medium induced by the fields.

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**Figure 9.59** Current probe using Faraday rotation of laser-generated light in a fiber-optic cable.

The main drawbacks of electro-optical diagnostics techniques is the cost of the equipment and the complex analyses necessary to unfold the data. In the case of the Faraday rotation measurement, birefringence in ordinary fiber-optic materials complicates the interpretation of the rotation, and extensive computations must be performed to determine the measured current. Electro-optical devices have some unique applications that justify the additional effort, such as the direct measurement of intense propagating microwave fields.